Hence $xy < 6$, and $y(x + 1) \equiv 0 \pmod{5}$. The solutions for $(x, y)$ are $(0, 10), (16, 0)$ and $(4, 1)$. Only the last choice gives integral values for $f_j$ and we then have by (6.5) and (7.11),

\[
\begin{pmatrix}
1 & 1 & 1 \\
10 & -5 & 1 \\
16 & 4 & -2
\end{pmatrix}, \quad \begin{pmatrix}
1 & 1 & 1 \\
1 & -1/2 & 1/10 \\
1 & 1/4 & -1/8
\end{pmatrix},
\]

\[
f_2 = \frac{27}{4.5} = 6,
\]

\[
f_3 = \frac{27}{1.35} = 20.
\]

The irreducible components have degrees 1, 6, 20, and the characters may be found by applying (6.10).

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**EQUAL SUMS OF LIKE POWERS**

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Let $s \geq 2$ and let $P(k, s)$ be the least value of $j$ such that the equations

\[
\sum_{i=1}^{j} a_i = \sum_{i=1}^{j} b_i = \cdots = \sum_{i=1}^{j} c_i \quad (1 \leq h \leq k)
\]

have a nontrivial solution in integers, that is, a solution in which no set $\{a_{iu}\}$ is a permutation of another set $\{a_{iv}\}$. It was remarked by Bastien [1] that $P(k, 2) \geq k + 1$ and this is true a fortiori for general $s$. The only upper bound for $P(k, s)$ for general $k$ and $s$ which I have found in the literature is due to Prouhet [5] who (in 1851) gave solutions of (1) with $j = s^k$, so that $P(k, s) \leq s^k$. He allocates each of the numbers $0, 1, \cdots, s^{k+1} - 1$ to the set $\{a_{iu}\}$ if the sum of its digits in the scale of $s$ is congruent to $u \pmod{s}$. Recently Lehmer [4] took $m_1, \cdots, m_{k+1}$ any $k + 1$ integers, let each of $b_1, \cdots, b_{k+1}$ run through

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1 Numbers in brackets refer to the bibliography at the end of the paper.
the values 0, 1, ..., \(s-1\) and allocated the number

(2) \[ b_1m_1 + \cdots + b_{k+1}m_{k+1} \]

to the set \(\{a_{tu}\}\) if \(\sum b_i = u \pmod{s}\). Lehmer's method provides a solution which may be trivial, though any set of \(m_i\) which makes the numbers (2) all different will certainly give a nontrivial solution. Prouhet's case, in which \(m_i = s^{i-1} \pmod{s}\), clearly does this.

The problem of determining \(P(k, 2)\) has received much attention. The inequality \(P(k, 2) \leq 2^k\), a particular case of Prouhet's result, was rediscovered in 1912 by Tarry [6] and by Escott [8]. This has since been improved [7] to

(3) \[ P(k, 2) \leq (k^2 + 4)/2. \]

In this note I find upper bounds for \(P(k, s)\) for general \(k\) independent of \(s\) and comparable with (3). Unlike Prouhet I do not find a particular solution of (1), but my method gives bounds for the \(a_i\). I cannot prove that \(P(k, s)\) is independent of \(s\), though I conjecture (somewhat more tentatively than for \(P(k, 2)\) in [7]) that \(P(k, s) = k+1\).

Various authors [2, 3] have shown that \(P(k, 2) = k+1\) for \(1 \leq k \leq 9\) and Gleden [3] proved that \(P(k, s) = k+1\) for \(k = 2, 3, 5\) and for all \(s\).

**Theorem 1.** \(P(k, s) \leq (k^2+k+2)/2\).

Let \(j = (k^2+k+2)/2\), \(n = (s-1)j!j^k\), \(1 \leq a_r \leq n\) \((1 \leq r \leq j)\), and

\[ l_h = a_1^h + \cdots + a_j^h. \]

Then \(j \leq l_h \leq jn^h\) and so there are at most

\[ \prod_{h=1}^{k} (jn^h - j + 1) < j^{2n^{(k+1)/2}} \]

different sets \(l_1, \cdots, l_h\). But there are \(n^j\) different sets \(a_1, \cdots, a_j\) and so more than \(j^{k} + j^{k-1} + \cdots + j = (s-1)j!\) sets \(a_1, \cdots, a_j\) associated with some one set \(l_1, \cdots, l_h\). Since the number of permutations of \(j\) objects among themselves is \(j!\), there are at least \(s\) sets \(a_1, \cdots, a_j\) which have the same \(l_1, \cdots, l_h\) and none of which is a permutation of any other. These provide a nontrivial solution of (1) with \(1 \leq a_{tu} \leq (s-1)j!j^h\).

**Theorem 2.** If \(k\) is odd, \(P(k, s) \leq (k^2+3)/2\).

For \(k = 1\) the theorem is trivial. Let \(k\) be odd, \(k \geq 3\), \(m = (k-1)/2\),
1948] EQUAL SUMS OF LIKE POWERS 757

\[ t = m(m + 1) + 1, \quad n = (s - 1)t!m, \quad 1 \leq a_r \leq n \quad (1 \leq r \leq t), \quad \text{and} \]

\[ L_h = a_1^{2h} + \cdots + a_t^{2h}. \]

Since \( t \leq L_h \leq tn^{2h} \), the number of different sets \( L_1, L_2, \cdots, L_m \) is at most

\[ \prod_{h=1}^{m} (tn^{2h} - t + 1) < t^m \prod_{h=1}^{m} n^{2h} = t^{m} n^{t-1}. \]

But there are \( n^t \) different sets \( a_1, \cdots, a_t \) and so more than \( t^{-m} n(t!)^{-1} = s - 1 \) sets \( a_1, \cdots, a_t \) which have the same \( L_1, \cdots, L_m \) and none of which is a permutation of any other. We take \( s \) of these sets, denote the numbers in them by \( a_1^{(u)}, \cdots, a_t^{(u)} \) \((1 \leq u \leq s)\) and put

\[ j = 2t = (k^2 + 3)/2, \]

\[ a_{iu} = n + 1 + a_i^{(u)} \quad (1 \leq i \leq t), \]

\[ a_{iu} = n + 1 - a_{i-t}^{(u)} \quad (t + 1 \leq i \leq j) \]

in (1). Since

\[ \sum_{i=1}^{t} a_{iu}^h = j(n + 1)^h + 2 \binom{h}{2} (n + 1)^{h-2} L_1 + 2 \binom{h}{4} (n + 1)^{h-4} L_2 + \cdots \]

and this is the same for all \( u \) when \( 1 \leq h \leq k \), we have a nontrivial solution of (1).

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