

changed, then the pitch p is unaltered.

III. If tangent planes be drawn through the rays of a congruence of any surface, the pitch p remains unaltered if the surface be deformed in any manner carrying the rays in its tangent planes.

Also is discussed the limiting value of the pitch, $dp/d\sigma$, which has interesting applications, in particular to the Anormalita of Levi-Civita.

This book is very readable and can be easily understood by any student who has had a first year course in Differential Geometry. In the opinion of the reviewer, this is a worthwhile addition to the library of any one who is interested in the classical theory of surfaces.

JOHN DECICCO

Methods of algebraic geometry. By W. V. D. Hodge and Daniel Pedoe. Cambridge University Press, 1947. 8+440 pp. \$6.50.

This work by two disciples of H. F. Baker naturally retains some of the flavor of the latter's *Principles of geometry*; but in keeping with the modern trend it is more algebraic and less geometrical. The spirit of the book is indicated by the fact that there is no mention of order or continuity. The first four of the nine chapters are concerned with algebraic preliminaries, chiefly in preparation for vol. II, and are so clear and concise that they would serve very well as an introduction to modern algebra, quite apart from their application to geometry. The topics treated in this part include groups, rings, integral domains, fields, matrices, determinants, algebraic extensions, and resultant-forms. The theory of linear dependence is developed without assuming commutativity of multiplication, and there is a neat algebraic treatment of partial derivatives and Jacobians.

Analytic geometry of projective n -space is taken up in Chapter V. A point of right-hand projective number space is defined as a set of right-hand equivalent $(n+1)$ -tuples of elements of a given field, not necessarily commutative; and right-hand projective space is defined as a set of elements which can be put in one-to-one correspondence with the points of such a number space by means of any one of a certain set of "allowable" coordinate systems. A linear subspace is defined as the set of points which are linearly dependent on $k+1$ linearly independent points; and the Propositions of Incidence follow readily. The notation of Möbius' barycentric calculus, as developed by Baker, arises naturally at this stage, and is used in proving Desargues' Theorem for coplanar triangles. Quadrangular constructions are given for the points $O+U(\alpha+\beta)$ and $O+U\alpha\beta$ as de-

rived from points O , U , $O+U$, $O+U\alpha$ and $O+U\beta$, and it is shown that Pappus' Theorem is equivalent to the commutative law $\alpha\beta=\beta\alpha$. In all such work, great care is taken to deal with all the special cases that may arise when points with different names coincide.

Chapter VI is an interlude on synthetic geometry, where linear spaces and incidence are primitive concepts, and the Propositions of Incidence are axioms. After constructing F. R. Moulton's non-Desarguesian plane geometry, the authors introduce Desargues' Theorem as an extra axiom to be used when the number of dimensions is only 2. A projectivity is defined as a product of perspectivities, and it is proved that any such chain of perspectivities can be reduced to as few as three. This is always an awkward piece of work, and the present version is as neat as any. (§§4 and 5 could perhaps have been shortened by first proving the uniqueness of the sixth point of the general quadrangular set.) The algebra of points on the projective line is developed in the manner of Veblen and Young, but with some improvements of detail. Homogeneous coordinates in n dimensions are introduced in a way that most ingeniously avoids any appeal to the Fundamental Theorem. Then various restrictions are considered. Pappus' Theorem is taken as a ninth axiom, and Desargues' Theorem is deduced from it as in Baker's *Introduction to plane geometry* (Cambridge, 1942, p. 26). Finite geometries are ruled out by a tenth axiom to the effect that a parabolic projectivity cannot be periodic.

Chapter VII contains an elegant exposition of Grassmann coordinates, generalizing the familiar properties of line-coordinates in 3-space. In the two final chapters, collineations and correlations are defined as linear transformations and are classified with characteristic thoroughness. The chapter on correlations is particularly valuable, as the authors have not shirked the formidable task of enumerating the various canonical forms for n dimensions. This last chapter also brings into prominence the skill of the compositors of the Cambridge University Press, which is to be congratulated on producing such a fine book in these difficult times.

H. S. M. COXETER

Set functions. By Hans Hahn and Arthur Rosenthal. The University of New Mexico Press, 1948. 9+324 pp. \$12.50.

When Hans Hahn died in 1934, he left manuscripts for the second volume of his treatise on real function theory. Arthur Rosenthal has now completed and edited that part of the work which deals with the theory of measure. This book gives a scholarly presentation of the foundations of the subject, taking account both of the theory of