

wide range of subjects that a comprehensive review is impossible. The topics range from *Philosophical foundations of probability* (Reichenbach) to *Biological association of insects* (Holloway) and *Wheat-bunt field trials* (Baker and Briggs). Of particular interest to pure mathematicians are survey papers by Doob, *Time series and harmonic analysis*, and by Feller, *On the theory of stochastic processes, with particular reference to applications*.

Although the preponderance of papers is devoted to statistics (general, descriptive, and mathematical) mention should be made of a paper on *Statistical mechanics and its applications to physics* (Lenzen) and on *Statistical study of the galactic star system* (Trumpler).

There is even a paper on *The place of statistics in the university* (Hotelling) followed by a discussion by five other authors.

In spite of the somewhat chaotic arrangement and an overwhelming battery of topics this volume is a tribute to the great vitality of statistics and statistical methods. It should prove a valuable addition to the rapidly growing statistical literature.

M. KAC

An essay toward a unified theory of special functions. By C. Truesdell. (Annals of Mathematics Studies, no. 18.) Princeton University Press, 1948. 4+182 pp. \$3.00.

It is very difficult to draw the line between mathematical physics and applied mathematics but this book shows that there does exist a definite and important difference between them. In mathematical physics many special functions such as the Legendre, Hermite, or Laguerre polynomials, Bessel or hypergeometric functions are used as tools to solve particular problems. As a consequence many properties of these functions and connections between them have been established. The author, as an applied mathematician, has posed the question of finding a unified approach to these different special functions so that from it most of the known properties could be found directly. The monograph under review gives an answer to this question.

The author found that many of the special functions, not only those previously mentioned but also such as the generalized Riemann zeta function, the incomplete gamma function, or the Poisson-Charlier polynomials, can be transformed into solutions of the equation

$$(1) \quad \frac{\partial F(z, \alpha)}{\partial z} = F(z, \alpha + 1).$$

A study of this equation shows that by simple techniques any solution

of (1) can be represented as a power series, a contour integral, or as a differential operator. For example, from a formal use of Taylor's theorem it follows that

$$(2) \quad F(z + y, \alpha) = \sum_0^{\infty} y^n F(z, \alpha + n)/n!.$$

This formula gives directly a generating function for the sequence $F(z, \alpha + n)/n!$, $n = 0, 1, 2, \dots$, while for $z = 0$ it gives a power series expansion of $F(y, \alpha)$.

As a result of these techniques the author is able to handle problems which could be treated only with difficulty by classical methods. For example, express the Laguerre polynomials in terms of the Legendre polynomials. The scope of the method and the ingenuity of the author are illustrated by the derivation of new, complicated formulas involving the special functions.

An essential point in the study of equation (1) is the proof of the following theorem: Given a bounded sequence $\phi(\alpha + n)$, $n = 0, 1, 2, \dots$, there exists one and only one solution of (1) such that

$$F(z_0, \alpha + n) = \phi(\alpha + n), \quad n = 0, 1, 2, \dots$$

This theorem is obtained as a special case of an existence theorem for a general vector difference-differential equation which the author proves. Once the uniqueness is known, it is easy to justify the formal applications such as those in (2).

The author concludes his monograph with some still unsolved questions. One such question is this:

What are the conditions to ensure the existence of a unique solution of the equation

$$(3) \quad \frac{\partial F(z, \alpha)}{\partial z} = F(z, \alpha - 1)$$

when $\alpha = \alpha_0 + n$, $n = 0, 1, 2, \dots$? If the answer to this were known, some important identities involving number-theoretic functions would be immediate consequences of (3).

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Non-linear problems in mechanics of continua. (Proceedings of Symposium in Applied Mathematics, vol. 1.) New York, American Mathematical Society, 1949. 8+219 pp. \$5.25.

This volume contains papers presented at the First Symposium in Applied Mathematics, held at Brown University, August 2-4, 1947.