RESEARCH PROBLEMS


If \( a_1 < a_2 < \cdots \) are positive integers, if \( C \) is compact, and if \( \sin a_n x \to 0 \) for all \( x \) in \( C \), prove that the convergence must actually be uniform. (Received July 18, 1954.)


If \( G \) and \( H \) are locally compact groups and \( f \) is a continuous function from \( G \) to \( H \), and if \( A \) is a Borel set in \( G \), prove that \( f(A) \) is measurable in the sense of Haar. (By Borel sets we mean the \( \sigma \)-ring generated by the compact sets.) The solution probably involves analytic sets; see, for example, Sierpinski, General topology, Toronto, 1934. (Received July 18, 1954.)

27. Walter Rudin: The cluster set of a meromorphic function.

If \( f \) is meromorphic in the open unit disc, then \( C(f) \), the cluster set of \( f \) in the large, is the set of all \( w \) (including \( \infty \)) for which there is a sequence \( \{z_n\} \) such that \( |z_n| < 1 \), \( |z_n| \to 1 \), and \( f(z_n) \to w \) as \( n \to \infty \). Every \( C(f) \) is a continuum, but not every continuum is a \( C(f) \). A counter-example is the set consisting of (a) a spiral, say \( r = \theta/(\pi + \theta) \), \( 0 \leq \theta < \infty \), (b) the unit circumference, (c) the interval \( 1 \leq x \leq 2 \), \( y = 0 \). The problem is to characterize, by means of intrinsic properties, those continua which are cluster sets in the above sense. Reference: Collingwood and Cartwright, Acta Math. vol. 87 (1952). (Received August 26, 1954.)


Let \( E \) be a set of the first category (and arbitrary measure) on the circumference \( C \) of the open unit disc \( U \), and let \( \phi \) be a continuous complex-valued function in \( U \). The proposer has recently proved that there exists an analytic function \( f \) in \( U \) such that, for every \( x \) in \( E \), (*) \( \lim_{n \to \infty} |f(rx) - \phi(rx)| = 0 \). Without further conditions on \( \phi \), the category condition cannot be dropped. The problem is to characterize those continuous functions \( \phi \) in \( U \) for which (*) may hold (i) for every \( x \) on \( C \), (ii) uniformly on \( C \), (iii) in some other norm. (Received August 26, 1954.)

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The Sixth Symposium in Applied Mathematics, p. 513. In the write-up of the Applied Mathematics Symposium in Santa Monica, California, in August, 1953, the name of the Consolidated Engineering Corporation of Pasadena, California, was inadvertently omitted from the list of computing facilities visited by members of the Symposium.

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David Nelson, Contraposition with strong negation. Preliminary report. Abstract 60-1-141. Line 5 of the abstract: for the second and third "D" read "N."