THE APRIL MEETING IN STANFORD

The five hundred fourteenth meeting of the American Mathematical Society was held at Stanford University, California, on Saturday, April 30. Attendance was approximately 100 including 86 members of the Society.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, Professor Irving Kaplansky of the University of Chicago and the University of California, Los Angeles delivered an address entitled *Operator algebras*. Professor Kaplansky was introduced by Professor J. L. Kelley. There were two sessions for contributed papers, over which Professors M. M. Schiffer and Randolph Church presided.

After the meetings, those attending were entertained at tea by the Stanford Department of Mathematics at the Stanford Alumni Hall.

Abstracts of papers presented at the meeting follow. The name of a paper presented by title is followed by "*t*." In the case of joint authorship, the name of the person presenting the paper is followed by (p).

Algebra and Theory of Numbers

579. Chen-Chung Chang: *A necessary and sufficient condition for an α-complete Boolean algebra to be an α-homomorphic image of an α-complete field of sets.*

Let α be any infinite cardinal. An α-complete Boolean algebra A is *α-representable* if it is an α-homomorphic image of an α-complete field of sets. For every $x \in A$, x has the property P_α, $P_\alpha(x)$, iff there exists a doubly indexed system of elements $\{a_{i,j}\}$, $i \in I$, $j \in J$, $\overline{\overline{I}} \leqq \alpha$, $\overline{\overline{J}} \leqq \alpha$, such that (1) $\prod_{j \in J} a_{i,j} = 0$ for each $i \in I$, and (2) for every function f on I to J the set of elements $\{a_{i,f(i)}; i \in I\}$ either contains x or contains a complementary pair. Let $I(\alpha, A) = \{x; x \in A \text{ and } P_\alpha(x)\}$. Theorems I–III below hold for α-complete Boolean algebras A. I. $I(\alpha, A)$ is an α-complete ideal in A and $A/I(\alpha, A)$ is α-representable. II. A is α-representable if, and only if, $I(\alpha, A) = \{0\}$. III. If J is any α-complete ideal in A, then A/J is α-representable if, and only if, $I(\alpha, A) \subseteq J$. IV. $I(\aleph_0, A) = \{0\}$ for any Boolean algebra A. II and IV immediately imply Loomis' result on representation of σ-complete Boolean algebras (Bull. Amer. Math. Soc. vol. 53 (1947) pp. 757–760). It follows from I–IV that the ideal $I(\alpha, A)$ may be regarded as the α-radical of the Boolean algebra A with respect to α-representation. It is known that there exist α-complete Boolean algebras A which are not α-representable (cf. Sikorski, Fund. Math. vol. 43 (1948) p. 247). Consequently, for these special algebras $I(\alpha, A) \neq \{0\}$. (Received March 7, 1955.)

580*t*. L. A. Kokoris: *Power-associative rings of characteristic two.*

It is proved that a commutative ring A whose characteristic is 2 is power-associative if $(x^2y)y = (y^2x)x$ and $x^{2^{n-1}}x^{2^{n-1}} = x^{2^n}$ for every x, y in A and every positive integer

n. When A is a commutative algebra over a field F of characteristic 2, the identity $(x^2y)y=(y^2x)x$ is automatically satisfied if F contains at least four elements. Examples are given to show that the hypotheses are necessary. (Received February 2, 1955.)

581*t*. P. J. McCarthy: *Some conditions under which genera of indefinite ternary quadratic forms exist which contain more than one class.*

In the course of research to determine necessary conditions for a genus of indefinite ternary quadratic forms to contain only one class, the following results have been obtained. For similar results see B. W. Jones and E. H. Hadlock, Proc. Amer. Math. Soc. vol. 4 (1953) pp. 539–543. Let f be a properly primitive, indefinite, ternary quadratic form with integral matrix A. Let the reciprocal form of f be properly primitive. Let Ω be the g.c.d. of the two-rowed minor determinants of A, and let the integer Δ be determined by $|A|=\Omega^2\Delta$. Let $\Omega=2^a\Omega_1^2p$ and $\Delta=-2^b\Delta_1^2p$, where Ω_1 and Δ_1 are odd, and p is an odd prime. Then the order of f contains a genus containing more than one class provided a and b are both even, $a+b\geqq 2$ when $p\equiv 5 \pmod 8$, and $a+b\geqq 4$ when $p\equiv 3 \pmod 4$. The same result holds true if $\Omega=2^a\Omega_1^2p$ and $\Delta=-2^b\Delta_1^2$, where Ω_1 and Δ_1 are odd, p is an odd prime, and p divides Δ_1. (Received March 7, 1955.)

582*t*. T. S. Motzkin: *Elimination theory for algebraic inequalities.*

Let $f(x)$, $x=(x_1, \cdots, x_n)$ be a polynomial with coefficients in a field K, and let $a(f)$, $b(f)$, and $c(f)$ be, respectively, the set of all x with $f=0$, $f\neq 0$ and (if K is the field R of reals) $f\geqq 0$. Let A, B, C and D be, respectively, a set obtainable by union and intersection from finitely many sets a; a and b; c; c and b. Let A_m, in n-space, be the projection of an A in $(m+n)$-space defined by polynomials $f(x, t)$, $t=(t_1, \cdots, t_m)$, and $A_{(m)}$ the same, using only polynomials homogeneous in t and excluding $t=0$. Let $\{A_m\}$ be the set of all A_m. Similarly define $\{A_{(m)}\}$, B_m, etc. Then for closed K, $\{A_{(m)}\}=\{A\}\neq\{A_m\}=\{B_{(m)}\}=\{B_m\}=B$; for $K=R$, $\{A\}$, $\{B\}$, $\{C\}$, $\{D\}$ are all different and $\{A_{(1)}\}=\{A\}$, $\{A_{(m+1)}\}=\{C_{(m)}\}=\{C\}$, $\{B_1\}=\{B\}$, $\{A_m\}=\{B_m\}=\{B_{(m+1)}\}=\{C_m\}=\{D_{(m)}\}=\{D_m\}=D$. (Received March 9, 1955.)

583. Morgan Ward: *Second order recurrences over p-adic fields.*

Let (w) be a linear recurrence of order two whose associated monic polynomial has coefficients in the rational p-adic field. Only the case when the coefficients are both units and the initial values w_0, w_1 integers is of interest. An index m for which w_m has a positive value is called a zero of (w). (w) may have no zeros; if it has, it is essentially determined by them. For each recurrence (w), there exists an essentially unique polar recurrence (v). The relationships between the zeros of (v) and (w) include as a special case Lucas' laws of apparitions and repetition for his numerical functions of the second order over the rational field. (Received March 7, 1955.)

ANALYSIS

584. J. O. Carter: *Some coefficient inequalities for schlicht functions.* Preliminary report.

This paper presents a straightforward method of obtaining a new upper bound on the absolute value of the fifth coefficient in the power series expansion of a schlicht analytic function. The method is based on the fact that if $f(z)=z+a_2z^2+a_3z^3+a_4z^4+a_5z^5+\cdots$ is a schlicht function, then $[f(z^{-3})]^{-1/3}=z+b_2z^{-2}+b_5z^{-5}+b_8z^{-8}+\cdots$ is also a schlicht function and $2|b_2|^2+5|b_5|^2+8|b_8|^2\leqq 1$, the classical area principle

[Bernardi, Duke Math. J. vol. 19 (1952) pp. 263–287]. The fifth coefficient a_5 is expressed as a linear combination of b_2, b_5, and b_8. The application of the above inequality leads to the upper bound $|a_5| \leqq 5.4$. (Received April 28, 1955.)

585. J. M. G. Fell: *Separable representations of purely infinite algebras.* Preliminary report.

A representation of an algebra A by bounded operators on a Hilbert space K is called separable if K is separable. This work is part of a program of finding what qualitative restrictions are imposed on the *-representations of a *-algebra by the assumption that the representation is separable. Let A be a purely infinite W^*-algebra (for terminology, see Kaplansky, Ann. of Math. vol. 53 (1951) pp. 235–249), and let R be a separable *-representation of A. It is shown that R is necessarily countably additive on the central projections of A. If in addition A is of type I, a complete description of R can be given: To each cardinal $n=0, 1, 2, \cdots, \aleph_0$, there is a unique central projection e_n in A such that (1) the e_n are pairwise orthogonal and have the unit element as their sum, and (2) R is the direct sum, over all n, of n copies of the projection of A into Ae_n. (Received February 25, 1955.)

586. R. D. Gordon: *A general solution of $dz=pdx+qdy$ with application to partial differential equations.*

A general solution of $dz\equiv pdx+qdy$ in terms of a certain class of variable transformations is described. By means of the resulting representations of x, y, z, p, q, the general solution of a general partial differential equation of second order (in two independent variables) is determined in terms of solutions of first order equations only. The solution readily fits standard conditions on an open boundary, subject to two differential inequalities associated with the boundary, the relation of which to the theory of characteristics has not been established. (Received February 2, 1955.)

587t. R. T. Prosser: *A general dimension function for W^* algebras.*

Let A be a W^* algebra, P the projections of A, Z the center of A, Γ the spectrum of Z, and σ the canonical representation of Z onto $C(\Gamma)$. For each nonzero $p \in P$, let $\mu(p)$ be the least upper bound of the cardinalities of the orthogonal families of equivalent nonzero projections $q \leqq p$, and put $\mu(0)=0$. Denote by L the positive reals, suitably extended by the adjunction of infinite cardinals, and topologized with the interval topology. A general dimension function for A is a mapping d which assigns to each $p \in P$ a continuous function on Γ to L, such that for all p, $q \in P$, (1) $pq=0$ implies $d(p+q)=d(p)+d(q)$, (2) $p\in Z$ implies $d(pq)=\sigma(p)d(q)$, (3) $|d(p)| \leqq \mu(p)$, (4) $d(p)=d(q)$ if and only if $p \sim q$. The existence and essential uniqueness of a general dimension function for any W^* algebra is established, and it is shown that much of the known structure of the algebra is carried by its dimension function. Similar results have recently been obtained by I. E. Segal. (Received March 9, 1955.)

588. R. T. Prosser: *On the structure of C^* algebras.*

Let A be a C^* algebra, A^* the dual space, and Ω the associated state space. By a left ideal of A is meant a norm closed left ideal; by a regular support of Ω is meant the intersection of Ω with a weak-*-closed supporting hyperplane in A^*. If J is a left ideal of A, then the set $\Xi=\{\omega: \omega \in \Omega \text{ and } \omega(a^*a)=0 \text{ for all } a \in J\}$ is a regular support of Ω, and conversely, if Ξ is a regular support of Ω, then the set $J=\{a: a\in A$ and

$\omega(a^*a)=0$ for all $\omega\in\Xi\}$ is a left ideal of A. It is shown that these relations generate a one-one inclusion-reversing correspondence between the left ideals of A and the regular supports of Ω. This correspondence relates the algebraic structure of A to the geometric structure of Ω, and has a number of interesting consequences. In particular, the maximal left ideals of A correspond to the extreme points of Ω. Since every support of Ω is generated by the extreme points it contains, it follows that every left ideal of A is the intersection of the maximal left ideals containing it. These results are easily verified when A is commutative. (Received March 9, 1955.)

Applied Mathematics

589. W. L. Hart (p) and T. S. Motzkin: *A Newton-Raphson method for systems of equations.*

Consider a system of k equations $f(x)=0$, where $f=(f_1,\cdots,f_k)$ are real functions of the n real variables $x=(x_1,\cdots,x_n)$, with continuous derivatives $f_{ij}=\partial f_i/\partial x_j$. For an approximation $x^{(h)}$, $h=0,1,\cdots$, to a solution obtain the next approximation $x^{(h+1)}=x^{(h)}+\rho_h\Delta x^{(h)}$, $\rho_h>0$, by setting $\Delta x^{(h)}=\sum_{i=1}^{k}\Delta_i x^{(h)}$, $\Delta_i x_j^{(h)}=-f_i(x^{(h)})f_{ij}(x^{(h)})\cdot(\sum_{j=1}^{n}f_{ij}^2(x^{(h)}))^{-1}$. If $f(x)=0$ is a homogeneous linear system of rank $r>1$ and $x^{(0)}$ is arbitrary, then $x^{(h)}$ tends geometrically to the solution nearest to $x^{(0)}$ provided the $\rho_h<2r/k$ are properly chosen, e.g., $\rho_h=2/k$. A best constant $\rho_h=\rho'$, in a certain sense, exists and $2/k\leqq\rho'\leqq 2(r-1)/k$. The same results hold for a general system $f(x)=0$ if the rank of the matrix (f_{ij}) is n, and if none of its rows is zero, at a solution $x=\alpha$, provided $x^{(0)}$ is sufficiently near α. For a linear system $f(x)=0$, consistent or not, with (f_{ij}) of rank r, under similar conditions for ρ, $x^{(h)}$ tends to x^*, where x^* is the point nearest $x^{(0)}$ of those points which minimize $\sum_{i=1}^{k}f_i^2(x)$, under the assumption that $\sum_{j=1}^{n}f_{ij}^2=1$. (Received March 9, 1955.)

590t. J. R. Jackson: *Notes on some scheduling problems.*

A theorem is established whereby the columns of a 3-row real matrix can be permuted in such a way as to minimize the maximum of certain "walks" through the matrix, over all permutations of the columns. The theorem is a slight generalization of some of the work of Selmer Johnson (*Optimal two- and three-stage production schedules with setup times included*, Rand Corp., 1953), and implies his results. It also yields solutions to the following scheduling problems: (1) To choose a schedule to minimize the overall production time for N items, the nth of which must be processed first by Machine A, then by Machine B_n, and finally by Machine C, with the requirement that Machines A and C process the items in the same order (Machine B_n is used only for Item n). (2) The two-machine problem of Johnson, with the possibility added that some items may start their processing on the second machine before they are completed on the first (under certain minor restrictions); as when the setting up of the second machine can be begun before the item to be processed is actually available. (Research supported by the Office of Naval Research). (Received February 14, 1955.)

Geometry

591. W. J. Firey: *On the densest packing of congruent, convex, material bodies.*

Material bodies are those which cannot have common interior points. Let $f(P)$ be the characteristic function of a distribution or packing of congruent convex material bodies. $K(r)$ is an admissible family of sets if: (a) $K(r)$ is convex for each r; (b) $K(r) \subset K(r')$ when $r<r'$; (c) given any point P, P is in $K(r)$ for r sufficiently large; (d) if $D(r)$ and $d(r)$ are the diameter and breadth of $K(r)$, there is a number M such that $D(r)/d(r) \leqq M$. Set $J(r)=\int_{K(r)}f(P)dP/V(r)$ ($V(r)$ is the volume of $K(r)$). The upper and lower limits of $J(r)$ are the upper and lower densities of the packing with respect to $K(r)$. A packing is absolutely densest with respect to $K(r)$ if no other packing of the same kind of bodies has a greater upper density with respect to $K(r)$, and if further the upper and lower densities of the packing with respect to $K(r)$ are equal. Theorem: There exists an absolutely densest packing of any particular kind of convex material body and its density is independent of the admissible family used to compute the density. (Received March 14, 1955.)

592*t*. I. M. Singer: *Transitive holonomy groups.*

Let $\mathcal{B}=[B, p, M, R_{2m}, R_{2m}]$ be a principal fiber bundle whose base space M is a manifold of dimension $2m$ and whose fiber R_{2m} is the rotation group acting on S^{2m-1}, the sphere of dimension $2m-1$. Let f be a map of M into the base space of the universal bundle relative to M which induces $\mathcal{B}$. Different conditions are given in terms of f insuring that the holonomy group of any connection on $\mathcal{B}$ acts transitively on S^{2m-1}. For example, if $2m=4k+2$ and if $f_*[\pi_{2m}(M)]$ is of infinite order, all holonomy groups will be transitive. The methods yield, among other things, a geometric proof that for $M=S^{2m}$, the holonomy group of any Riemannian connection is always R_{2m}. If $\mathcal{B}$ is the bundle of orthogonal frames of M relative to a Riemannian geometry on M, the theorems give a topological proof of only a part of M. Berger's extensive results [C. R. Acad. Sci. Paris vol. 237 (1953) pp. 472–473 and 1306–1308; vol. 238 (1954) pp. 985–986], namely when M is never reducible or symmetric. (Received March 9, 1955.)

Statistics and Probability

593. Z. W. Birnbaum (p) and O. M. Klose: *Bounds for the variance of the U-statistic in terms of $p=\text{Prob}\{Y<X\}$.*

Let X_i, $i=1, 2, \cdots, m$, and Y_j, $j=1, 2, \cdots, n$, be samples of independent random variables X and Y, with distribution functions $F(x)$ and $G(y)$, and let $U=$number of pairs (X_i, Y_j) such that $X_i>Y_j$. Then $mn\{p(1-p)+[2(2p)^{3/2}/3-2p^2]\cdot[\min(m, n)-1]\}\leqq\sigma^2(U)\leqq mnp(1-p)\operatorname{Max}(m, n)$ and both bounds are sharp for $m=n$ (upper bound obtained by D. Van Dantzig). If $F(s)\geqq G(s)$, i.e. X is "stochastically smaller" than Y, then the upper bound can be replaced by $mn\{(m+n+1)+6(n-m)h^2+4(2m-n-1)h^3-3(m+n-1)h^4\}/12$ where $h=(1-2p)^{1/2}$. This bound is sharp for any $m\leqq n$. (Received March 10, 1955.)

594. Edwin Hewitt (p) and H. S. Zuckerman: *Arithmetic and limit theorems for a class of random variables.*

Let G be a finite commutative semigroup for which there exists an integer $m>1$ with the property that $x^{m+1}=x$ for all $x\in G$. Let $\mathfrak{P}(G)$ be the set of all non-negative functions f on G with $\sum_{x\in G}f(x)=1$. $\mathfrak{P}(G)$ is closed under the convolution $f*g(x)=\sum_{uv=x}f(u)g(v)$, and also under convex addition. $\mathfrak{P}(G)$ consists of all probability distributions of random variables with values in G. $\mathfrak{P}(G)$ has a unit iff G has a unit. $\mathfrak{P}(G)$ always has a zero. An element $f\in\mathfrak{P}(G)$ is idempotent iff f is uniformly distributed on a subgroup of G. An element $f\in\mathfrak{P}(G)$ is the limit of a product

$g_1 * g_2 * \cdots * g_k$ where the g_k's are nontrivial iff f is positive exactly on a union of cosets of a subsemigroup D of G, where $o(D) > 1$ and D has a unit. Several examples are worked out in detail. (Received February 28, 1955.)

Topology

595. James Dugundji and E. A. Michael (p): *Local and uniformly local topological properties.*

It is shown that certain local properties of a metrizable space, such as local connectedness, local contractibility, or LC^n, hold uniformly with a properly chosen metric. An analogous result is proved for properties such as being an ANR. (Received March 11, 1955.)

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