

pendent on the particular mathematical form used to express them. Indeed, if a 4-dimensional treatment is employed the vorticity is no longer a vector but is described by three of the components of a second-rank antisymmetrical tensor. Evidently Truesdell means that his particular formalism, in terms of 3-vectors, is valid only in three dimensions which is no doubt the case; but over-emphasis on this point of view leads to remarks such as that on p. 77, relative to Lagrange's acceleration formula. Whilst it is true that the particular formula (38.2) holds only in 3 dimensions, it is also true that a corresponding formula, involving the vorticity tensor, holds in 4 dimensions. Thus it does not appear to be correct to state without qualifications that "for the existence of Lagrange's formula it is requisite that the number of dimensions be three."

But when all has been said, one important fact emerges: this book is a valuable compendium of results that every expert in hydrodynamics, gas-dynamics or dynamical meteorology will want to keep by his side and refer to frequently.

G. C. McVITTIE

Introduction to integral geometry. By L. A. Santaló. Paris, Hermann, 1953. 127 pp. 1500 fr.

Integral geometry is the name given by Blaschke to a branch of geometry which originated with problems on geometrical probabilities and deals with relations between measures of geometrical figures. One should perhaps consider as its father the English geometer W. F. Crofton, while Poincaré left an important mark by introducing the kinematic measure (and by writing a book on geometrical probabilities).¹ In 1934 Blaschke and his coworkers started a series of papers on the subject. The author of this book made some of the most beautiful contributions to it in that period and, in the twenty years which followed, has consistently added new results to it. It is therefore gratifying that a book by the author now exists in the literature. The book is elementary in nature. Its first two parts presuppose only some knowledge of calculus and the last part some projective geometry and a little more maturity.

Part I, on the metric integral geometry of the plane, studies the densities of points, lines, pairs of points, etc. and culminates with Poincaré's kinematic density and Blaschke's kinematic formula. One of the most interesting applications is the author's proof of the

¹ As a matter of historic interest mention should be made of a set of lecture notes by G. Herglotz, which introduced many of the early workers into the subject.

isoperimetric inequality in the plane. In spite of the elementary nature of plane geometry, this is still the most fascinating chapter of the book. The presentation is informal and lucid. The author has certainly succeeded in leading up effortlessly to many beautiful results. One should, however, not be led to think that everything in this subject is so simple. The author has purposely neglected the set-theoretic difficulties. Otherwise even the proof of the simplest of Crofton's formulas would have taken pages, while a proof of the kinematic formula for fairly general domains would be quite complicated.

Part II generalizes the results of plane integral geometry to surfaces. The generalization is complete in the case of surfaces of constant curvature. In the general case a density for geodesics and the kinematic density are defined. To those familiar with modern notions in differential geometry it would be conceptually simpler to define the kinematic density as one in the circle bundle of unit tangent vectors and the density for geodesics as one in a leaved structure (structure feuilletée of Ehresmann). The kinematic density on a surface is probably an important notion. It may be of interest to mention that it was used by E. Hopf to prove his theorem that, if a closed surface has no conjugate points, its Euler characteristic is ≤ 0 (Proc. Nat. Acad. Sci. U. S. A. vol 34 (1948) pp. 47-51).

Part III is concerned with integral geometry in a general homogeneous space and constitutes one half of the book. It begins with a brief introduction to the local theory of Lie groups, following E. Cartan. Using this general theory, a necessary and sufficient condition is established for the existence of an invariant density when the homogeneous space is acted on by a connected group. The general results are then applied to special cases: the Cayley plane, affine and projective geometry. Perhaps the most interesting result is the interpretation of Siegel's proof of the Minkowski-Hlawka theorem as a result of integral geometry. It seems to the reviewer that immediate further results are to be found in the case when the isotropy group of the homogeneous space is compact.

The question of studying the relations between measures in homogeneous spaces with the same group is a natural one, be it the Euclidean plane and the space of its lines or the affine plane and the space of its lattices in the sense of the geometry of numbers. The final word on these questions has not been said. We may hope that the publication of this book will contribute to further progress in this neglected but highly fascinating branch of geometry.

S. S. CHERN