Essays like this one by Professor Truesdell can help to overcome this lack of information. It is not just a question of historical piety and correct assignment of priority. Euler actually is good reading, and we must consider Professor Truesdell's introduction as an invitation to read him, like Christopher Morley's introduction to Shakespeare. After all, as Jacobi already said: "Today it is quite impossible to swallow a single line by D'Alembert, while we still can read most of Euler's works with delight."

D. J. STRUIK

Algèbre locale. By P. Samuel. (Mémorial des Sciences Mathématiques, no. 123.) Paris, Gauthier-Villars, 1953. 76 pp. 950 fr.

The object of this monograph is a presentation of the theory of local rings and their generalizations, the semilocal rings and the M-adic rings. Those aspects of the theory are dealt with which are valid in rings of arbitrary dimension. Thus the special properties of one-dimensional rings such as rings of p-adic integers or of power series in one variable are not included.

All rings considered are commutative and have an identity element. *Local rings* were introduced some fifteen years ago by Krull; they are Noetherian rings with a single maximal ideal. More generally, a *semilocal ring*, in the sense of Chevalley, is a Noetherian ring A with only a finite number of maximal ideals. If M is their intersection, then $\bigcap_{n=1}^{\infty} M^n=(0)$, and the sequence of ideals $\{M^n\}$ defines a Hausdorff topology on A. More generally, a Noetherian topological ring in which the topology is Hausdorff and is defined by the powers M^n of some ideal M is called *M-adic*. A *Zariski ring* is an M-adic ring in which every ideal is a closed set. M-adic rings and Zariski rings were introduced by Zariski (who, however, called the latter type *generalized semilocal*). The semilocal rings of Chevalley are Zariski rings, as are all complete M-adic rings. The more elementary properties of these rings are considered in Chapter I. Here are discussed their completions, homomorphisms, quotient rings, direct decompositions, and finite extensions.

In Chapter II we are concerned with a semilocal ring A and a defining ideal V of A—that is, an ideal V in A such that $M^t\subset V\subset M$, t being some integer and M the product of the maximal ideals of A. It is then proved that the length of A/V^n as an A-module is a polynomial of $P_V(n)$ for n sufficiently large. The degree d of this polynomial is independent of V and is, in fact, the minimum number of generators in any defining ideal. It is called the dimension of A and thus coincides with the notion of dimension of a local ring in the sense of Chevalley and Krull. The polynomial $d!P_V(n)$ has integer coefficients and its leading coefficient $e(V)$ is called the *multiplicity* of V. This generalizes a similar notion of Chevalley, who used it to define intersection multiplicities in algebraic geometry. The contents of this chapter, which include various properties of $P_V(n)$ and of $e(V)$, constitute a generalization and simplification of some previous results of the author.

Chapter III is devoted to the study of *geometric local rings*, a class of rings which includes the local rings of points on algebraic variety. Let K be a field and let $N(n, K)$ be the quotient ring of the polynomial ring $K[X_1, \cdots, X_n]$ with respect to the ideal $(X_1, \cdots, X_n)$. The geometric local rings are the members of the smallest class of rings containing all $N(n, K)$ and closed under the operations of completion and formation of quotient ring or residue class ring with respect to a prime ideal. These rings G are more tractable than local rings in general for they are rings with nucleus—i.e., they contain a nucleus N and an intermediate ring B $(N\subset B\subset G)$ such that B is finite over N, G is a quotient ring of B with respect to a maximal ideal. A *nucleus* is hereby a ring of the form $N(n, K)$ or $N(n, m, K)$, the latter being the quotient ring of the formal power series ring $K\{X_1, \cdots, X_n\}$ with respect to the ideal $(X_{m+1}, \cdots, X_n)$. An important relation exists between the multiplicities in a geometric local ring A and its completion A^*. Namely, if Q is primary in A and Q^* is a primary component of A^*Q, then Q and Q^* have the same length. It follows that if Q is prime then A^*Q is an intersection of prime ideals. Moreover, if P is the radical of the primary ideal Q, and if P^* is a prime divisor of A^*P, then $e(A_PQ)=e(A^*_{P^*}Q)$. This "theorem of transition" allows passage in proofs from a ring to its completion, for example in the proof of the following theorem. Let A be a geometric local ring, Q an ideal generated by a set of parameters, V an ideal generated by a subset of these parameters, and $\{P_i\}$ the minimal prime divisors of V; then

$$e(Q) = \sum_i e((Q + P_i)/P_i)e(A_{P_i}V).$$

This is the associativity formula, an all-important one for the theory of multiplicities. The contents of this chapter are due to Chevalley, with some improvements by the author. In the definition of nucleus, Chevalley and the present book assume for certain technical reasons that $[K:K^P]$ is finite, P being the characteristic. However, Nagata has shown that this restriction can be eliminated [Sûgaku, vol. 5 (1954) pp. 229–238 (in Japanese)].

The material of Chapter IV, on the structure of complete local rings, is due to the reviewer. Let A be a complete local ring, M its maximal ideal, ϕ the canonical homomorphism of A onto A/M. If A and A/M have equal characteristics p, then A contains a field K such that $\phi(K)=A/M$. If $p=0$, then this can be readily derived from "Hensel's Lemma," the statement of which for complete local rings is just what one expects; in the more difficult case $p>0$, the proof follows from the lifting theorem stated below. (A particularly simple proof in this equal-characteristic case has been recently given by A. Geddes [J. London Math. Soc. vol. 29 (1954) pp. 334–341].) Suppose now that A/M has characteristic $p>0$ (which is certainly the case if this characteristic is different from that of A, on which we make no assumption), let B be a complete discrete valuation ring of characteristic zero whose maximal ideal is generated by p, and assume τ a homomorphism of B onto A/M (existence of B and τ is well known). The lifting theorem then asserts the existence of a homomorphism σ of B into A such that $\phi\sigma=\tau$. From this can be deduced that every complete local ring is the homomorphic image of a power series ring over K or B, and also is (in the absence of zero divisors) a finite module over such a power series ring. Some further theorems on structure are proved and some applications made to the ideal theory in complete local rings.

In Chapter V, it is proved that if a ring with nucleus lacks zero divisors and is integrally closed then its completion also possesses these properties. This is a generalization of Zariski's theorem on the analytical irreducibility and analytical normality of normal varieties. A form of the Weierstrass preparation theorem is given and with its aid unique prime factorization is proved in the power series rings described in the preceding paragraph.

In Chapter VI there is defined a Kronecker product for two M-adic rings. Some other questions are briefly considered.

I. S. Cohen

Editorial note: The preceding review was found among Professor Cohen's papers after his death. Attached notes indicate that a final paragraph was to have mentioned the historical notes at the end of the chapters; the extensive bibliography (98 items); and the existence of a number of misprints, some merely typographical, others actual mistakes (pp. 22, 35, 37, 51). The mistake which Cohen considered most serious appears to be on p. 35, fourth line from the bottom.

Funzioni ipergeometriche confluenti. By F. G. Tricomi. Rome, Cremonese, 1954. 16+312 pp. 3500 Lire.

This book on confluent hypergeometric functions differs quite con-