various contributions to the unsolved problem above and a neat characterization of the class of functionals generated by V, F, and M in terms of functional properties alone. In general, of course, the book is expository, developing in particular Blaschke's idea of introducing a metric into $\mathfrak{C}$. However, the thoroughness with which this idea is exploited is due to the author, who throughout has kept a nice balance between his abstract approach and the concrete results achieved.

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Méthodes d'algèbre abstraite en géometrie algébrique. By P. Samuel. Berlin, Springer, 1955. 9+133 pp. 23.60 DM.

The goal of the present work is, according to the author, "to give as complete an exposition of the foundations of abstract algebraic geometry as is possible," and to be useful to the practitioner ("l'usager" as Samuel calls him). Actually the main use of this book will be found as a handbook for one who wishes a less abrupt and difficult introduction to the abstract methods of algebraic geometry than is afforded by Weil's *Foundations* (which, it is too often forgotten, was not meant as an introduction). This latter book begins with three arduous chapters on pure algebra, whose use does not become apparent until much further in the book. Such a barrier does not exist in Samuel's exposition, because he assumes known all the needed basic algebra, or rather refers as he goes along to an appendix containing purely algebraic basic results, or references to those in the literature. (This of course could not have been done by Weil, for the good reason that most of these results were not in the literature at the time.) Samuel proceeds immediately with the geometric language and hence the reader's first contact with abstract methods is reasonably soft.

The book is divided into two parts. The first one gives the general theory of algebraic varieties, defined as either affine or projective varieties. It begins with the notion of algebraic set (set of zeros of polynomial ideals), union and intersection of these, and continues with the notion of dimension, generic points, products, projections, correspondences, rational and birational maps. The deepest theorem in this part asserts that every component in the intersection of two varieties V and W of dimension r and s in projective n space has dimension $\geqq r+s-n$. Elimination Theory is discussed as a special case of the theory of projections (as it should be) and is derived elegantly from the basic and elementary theorem on the extension of specializations. So is the Hilbert Nullstellensatz.

There is a section on properties which hold almost everywhere (i.e. at least on the complement of some proper algebraic subset of a given variety), followed by a section on Chow coordinates. The notion of normality is also introduced and two useful results are proved: a point on a variety V/k which is k-normal remains normal over a separably generated extension of k, and the product of two normal points is normal on the product variety.

The second part deals mostly with local problems. It begins with a further discussion of normal points (see further comments below concerning ZMT) and with the theory of simple points. It is mostly devoted to an account of the intersection theory. The proofs are sketched (but can be read without difficulty) and follow more or less closely those given by the author in his thesis. They belong to what Severi calls the static method (local rings and multiplicity of a primary ideal being the fundamental notions around which all others revolve).

The enumeration which precedes will suffice to show that the book contains a wealth of basic material, and as we have said earlier, will be of great use to one desiring to learn this material. (It will also suffice to show that the expert will find little in the book which he does not know already.)

Granting the author's point of view, which was to write a complete foundational book on the abstract methods of algebraic geometry, the reviewer believes nevertheless that the book has the serious defect of not being complete. We shall illustrate this by some specific examples.

a. *Abstract varieties.* No mention is made of abstract varieties. Of course no one wishes to ban projective varieties, and we are not promoting a competition between abstract and projective varieties. One should note however that notions like products, intersections, projections, are all intrinsic and do not require an embedding in projective space. (No differential geometer would define the product of two differentiable manifolds by first requiring that they be imbedded in Euclidean space.) More seriously, certain problems are best treated, and others can only be treated by dealing directly with abstract varieties. (For instance, algebraic groups, homogeneous spaces, fibre spaces. It is of course useful, and highly desirable, to have an embedding theorem, because we gain thereby an additional tool for studying them. In other topics (we are thinking particularly of Serre's theory of coherent sheaves) the main theorems are all concerned with projective varieties, but the theory of abstract varieties is a technical necessity, partly because the embedding changes so often, that one needs some underlying structure which does not change.

b. *Properties holding almost everywhere.* The Zariski topology should have been defined in the section dealing with properties holding almost everywhere. The closed sets on a variety are its algebraic subsets, and a set is k-closed if it can be defined by equations with coefficients in k. Furthermore, one should note that in practice it is usually not sufficient to know that a property holds almost everywhere (on a set containing a nonempty open set) but one must know that the property holds exactly on an open set. Many of Samuel's properties which are stated to hold almost everywhere in fact hold on an open set. For precise results in this direction, see for instance Weil's *Critères d'equivalence*, Math. Ann. (1954) referred to as *Crequ.* and also the Appendix to his *Groups of transformations*, Amer. J. Math. (1955) referred to as *GT*.

c. *Chow coordinates*. The treatment here given follows closely the original one given by Chow and van der Waerden. One notable addition is Chow's theorem that for a divisor on a variety, the smallest field of rationality coincides with the field generated by the Chow coordinates. Although this conclusion does not remain valid for an arbitrary cycle (in characteristic p), it should have been remarked that it does if the coefficients of the components of the cycle are not divisible by p. See for instance GT, Appendix. Other useful properties not included here are given in Crequ. It would be highly desirable to have once for all a *complete* list of the properties of these coordinates, and of the theorems which relate them with the theory of intersection and specialization of cycles. The results given here are only fragmentary.

d. *Normalization and ZMT*. We come here to one of the main omissions, that of Zariski's Main Theorem on birational correspondences (referred to as ZMT). Let $T: V\rightarrow W$ be a rational map having the property that the total image of a point P normal on V has one component consisting of a single point Q. Then this total image is exactly Q, and T is well defined at P. Only a trivial special case of ZMT is stated in II, §§3, 4. This is all the worse because a complete proof of it could have been given easily with the techniques used in this second part of the book. The theorem concerning the analytical irreducibility of a variety at a normal point is proved in the adjoining section, and one of its most interesting applications is precisely ZMT itself.

Apropos of normalization, we note in passing that the statement on page 125 that the Italian geometers had the notion is quite misleading. They took as definition the property that the system of hyperplane sections is complete. This is a global property, depending on the embedding, whereas Zariski's notion of a normal point is local, depending on the local ring of the point.

e. *Intersection theory*. A strong warning should have been given here (but was not) that the definition of cycles does not coincide with that of Weil. Weil defines an s-cycle on a given variety V to be an element of the free abelian group generated by the simple subvarieties of V, of dimension s. For Samuel, a projective space P_n is fixed, and his s-cycles are cycles on P_n in the Weil sense. A cycle on a variety V in Samuel's terminology simply means that the point set of the cycle is carried by V. This difference in the basic definitions entails a technical difference in the results of the intersection theory. Samuel's is admittedly more general, and at least once has been used in a more advanced investigation (Néron's thesis), but its use is seriously impaired by the incompleteness of his list of theorems. This incompleteness is most striking in those parts dealing with divisors of functions. Very few of the results stated by Weil in F-VIII are included in Samuel's book. Some of them which look formally alike actually differ, as for instance the theorem relating the divisor of a function f on a variety V and the divisor of its induced function on a subvariety W. This is due to the fact that Samuel's definitions allow components of the divisor of f to be singular on V. One consequence of this is that Theorem 10 of F-VIII_3 does not remain valid with Samuel's definition (to cite only one example). (It is the theorem stating essentially that when a divisor X is rational over k, then the space of functions whose divisors are $>X$ has a basis defined over k.) Other examples of useful theorems not included by Samuel are those dealing with the relation between the divisors of a function and its norm over a subfield (F-VIII_2 Th. 7) and functions on product varieties (F-VIII_2 Th. 1 and its corollaries).

The principal avowed purpose of *Foundations* was to furnish the working algebraic geometer with a *complete* and secure encyclopedia of basic "obvious" results, and to spare him the task of having to reconsider constantly foundational lemmas in the course of a more advanced investigation. The above comments show clearly that Samuel's book cannot be used in a similar capacity. In the preceding list of topics which might have been more completely treated, we have even limited ourselves to those which fit in directly with Samuel's own choice of general material for his book, and whose inclusion would not have materially increased the length of the book, but would have greatly increased its value.

On the other hand, we can think of a number of other topics which might also be characterized as belonging to the foundations of abstract algebraic geometry, and might have been included. For instance, linear systems and Bertini's theorems, and differential forms. These would have found a natural place in the book, at the cost of a quite small increase in size, and their inclusion would have been