817. G. T. Whyburn: Dimension and nondensity preservation of mappings.

The property of a mapping \( f(X) = Y \) to preserve nondensity for compact subsets of \( X \) is characterized in terms of the lightness kernel \( L_f \) of \( f \) consisting of all \( x \in X \) such that \( f^{-1}(x) \) is totally disconnected. These results are applied to show that if \( w = f(z) \) is continuous in a region \( X \) of the \( z \)-plane and differentiable at all points of a dense set \( f^{-1}(Y) \) in \( X \) which is the inverse of an open subset \( Y \) of \( Y = f(X) \), then \( \dim f(K) \leq 1 \) for each compact set \( K \) in \( X \) of dimension \( \leq 1 \). Under the same conditions it is shown that \( f \) is strongly quasi-open. (Received July 5, 1956.)


Let \( X \) be a topological space and let \( (X, S, \mu) \) be an associated measure space. Let \( f(x) \) be non-negative and measurable \((S) \). Define the measure \( v \) on \( S \) by means of the formula:

\[ v(E) = \int_E f(x) \, d\mu(x), \]

for all \( E \) belonging to \( S \). Using the ancient techniques of measure theory, one can easily establish the following remarks: (1) If \( \mu \) is inner regular, so also is \( v \). (2) If \( \mu \) is outer regular, then \( v \) is outer regular if and only if for every set \( E \) on which \( f \) is integrable: (i) There exists a measurable open set containing \( E \) on which \( f \) is also integrable. (ii) There exists, for every pair of positive numbers \( \epsilon, \delta \), a measurable open set \( U \) containing \( E - N(f) \) such that \( \mu(\{x: f(x) > \delta\} \cap U) < \epsilon \). (Received July 5, 1956.)

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RESEARCH PROBLEMS

15. Richard Bellman: Approximation theory.

Consider the differential equation (1) \( dx/dt = \Phi(x) \), \( x(0) = c \) where \( \Phi(x) \) is a continuous function of \( x \) satisfying additional conditions ensuring the existence of a unique solution over the interval \( 0 \leq t \leq T \) for \( a \leq c \leq b \).

Let (2) \( dy/dt = \sum_{k=0}^{K} a_k(c, t)y^k \), \( y(0) = c \), represent an approximation to (1) where the coefficients \( a_k(c, t) \) are functions to be determined so as to minimize one of the following functionals: (3) (a) \( J_1 = \int_0^T |x - y| \, dt \), (b) \( J_2 = \int_0^T (x - y)^2 \, dt \), (c) \( J_3 = \max_{0 \leq t \leq T} \max_{a \leq c \leq b} |x - y| \), (d) \( J_4 = \max_{0 \leq t \leq T} \int_0^T (x - y)^2 \, dt \), (e) \( J_5 = \int_0^T \left[ \sum_{k=0}^{K} (x - y)^2 \right] \, dc \) under the following alternatives: (4) (a) \( a_k(c, t) \) depends only upon \( c \), for \( k = 0, 1, \ldots, K \). (b) \( a_k(c, t) \) are polynomials of degree \( M \) in \( t \) with coefficients dependent upon \( c \). (The case \( K = 1 \) is the most interesting.)

Consider the analogous problem for systems of the form (5) \( dx_i/dt = \Phi_i(x_1, x_2, \ldots, x_n) \), \( x_i(0) = c_i, i = 1, 2, \ldots, n \), in the particular case where the approximating equation is (6) \( dy_i/dt = a_i(c) + \sum_{j=1}^{n} b_{ij}(c)y_j(t), y_i(0) = c_i \), and we wish to minimize the functional (7) \( \int_0^T \cdots \int_0^T \left[ \sum_{i=1}^{n} (x_i - y_i)^2 \right] \, dt \, dc_i \). (Received August 17, 1956.)