THE OCTOBER MEETING IN CAMBRIDGE

The five hundred twenty-seventh meeting of the American Mathematical Society was held at the Massachusetts Institute of Technology on Saturday, October 27, 1956, in conjunction with a meeting of the Society for Industrial and Applied Mathematics. About 240 persons attended, including 200 members of the Society.

An address entitled *Coverings of algebraic varieties* was presented by Professor J-P. Serre of the Collège de France and the Institute for Advanced Study by invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings. Professor Richard Brauer presided.

Sessions for contributed papers were held in the morning and afternoon, with Professors A. P. Calderón, J. B. Diaz, Dr. A. P. Mattuck, Professors L. N. Howard, D. J. Struik presiding.

Abstracts of the papers presented follow. Where a paper has more than one author, that author whose name is followed by "(p)" presented it. Those papers with "*t*" following their numbers were presented by title. Dr. Edwards was introduced by Mr. D. G. Kendall, and Mr. Manly by Mr. R. R. Christensen.

Algebra and Theory of Numbers

1. A. T. Brauer: *On the theorems of Ledermann and Ostrowski on positive matrices.*

Let A be a positive matrix of order n, and R_ν the sum of the elements of the νth row. We set $\max R_\nu = R$ and $\min R_\nu = r$. A well known theorem of Frobenius [Sitzungsberichte Preussische Akademie der Wissenschaften (1998) pp. 471–476 and (1909) pp. 514–518] states that A has a greatest positive characteristic root ω. Moreover, we have $R \geqq \omega \geqq r$. For $R = r$, these inequalities cannot be improved. Assume now that $R > r$. We denote the smallest element of A by m. W. Ledermann [J. London Math. Soc. vol. 25 (1950) pp. 265–268] and A. Ostrowski [J. London Math. Soc. vol. 27 (1952) pp. 253–256] obtained better bounds which are functions of R, r, and m. In this paper the best possible bounds of this kind and some similar results are obtained. (Received September 10, 1956.)

2*t*. Barron Brainerd: *On a class of lattice-ordered rings*, I.

An *F-ring* R is a σ-complete vector lattice which is also a commutative algebra with unit $1 > 0$ in which $x \geqq 0$, $y \geqq 0 \Rightarrow xy \geqq 0$ and $x \wedge 1 = 0 \Rightarrow x = 0$. Let R be regular, that is, for each $x \in R$ there exists $x^0 \in R$ such that $xx^0x = x$. The sub-F-ring $\overline{R} = \{x \mid x \in R, |x| \leqq \lambda \cdot 1 \text{ for some real } \lambda\}$ is a real Banach algebra. A (ring) ideal J of an F-ring is *closed* if $a_n \in J$ $(n \geqq 1)$ and $\sup_n a_n$ exists imply that $\sup_n a_n$ belongs to J. A maximal ideal M of R is closed if and only if $R - M$ is isomorphic to the real field. There is a one-to-one correspondence between the maximal ideals of R and those of $\overline{R}$. For a maximal ideal M of R to be closed, it is necessary and sufficient that $M \cap \overline{R}$ be a

maximal ideal of $\overline{R}$. There is a one-to-one correspondence between the closed maximal ideals of R and those of $\overline{R}$. If the intersection of the closed maximal ideals of R is zero, then R is (F-ring) isomorphic to the F-ring of all $(\Phi, \mathfrak{I})$-measurable functions for a pair $(\Phi, \mathfrak{I})$ which is defined in terms of R. A real function f on Φ is $(\Phi, \mathfrak{I})$-*measurable* provided $\{x|f(x)\leqq\lambda\}$ belongs to the σ-algebra $\mathfrak{I}$ for each real λ. These results are obtained by exploiting the concept of projector as defined by S. Kakutani [Ann. of Math. vol. 42 (1941) pp. 223–237]. (Received August 8, 1956.)

3*t*. Barron Brainerd: *On a class of lattice-ordered rings*, II.

The terminology and notions of I are used. The set I of idempotents of an F-ring A forms a σ-complete Boolean algebra (σ-B.A.) under the lattice operations of A. I is the *idempotent algebra* (i.a.) of A. Every σ-B.A. I is the i.a. of a regular F-ring $R(I)$. A result of Sikorski [Fund. Math. vol. 35 (1949) pp. 247–258] is used to construct $R(I)$ which is a uniquely determined homomorph of the F-ring of all $(\Phi, \mathfrak{I})$-measurable functions for a pair $(\Phi, \mathfrak{I})$ defined in terms of I. Olmsted [Trans. Amer. Math. Soc. vol. 51 (1942) pp. 164–193] has constructed for each σ-B.A. I an F-ring $\Omega(I)$ with I as its i.a. If R is a regular F-ring with i.a. I, then R is isomorphic to both $\Omega(I)$ and $R(I)$. Every F-ring having I as i.a. is isomorphic to a sub-F-ring of $R(I)$. Let B be a semi-ordered ring with unit $1>0$ which is archimedian and in which $\inf_{\gamma\in\Gamma} a_\gamma=0$ and $b\geqq 0\Rightarrow\inf_{\gamma\in\Gamma} a_\gamma b=0$. From results of Nakano [*Modern spectral theory*, Chapter V], it follows that B can be imbedded in an F-ring R. Therefore there is a σ-B.A. I such that B is isomorphic to a sub-semi-ordered ring of $R(I)$. (Received August 8, 1956.)

4. Walter Feit: *Arithmetic in algebras.* Preliminary report.

Let A be a semi-simple algebra over a field K, D an integral domain whose quotient field is K. Pick a basis of A such that the D module generated by this basis forms a ring R with a unit element. The ring R can then be written as a direct sum of a finite number of indecomposable left ideals. A left ideal is a direct summand if and only if it contains an idempotent element. If K is a p-adic field and D the ring of integers in K, then there is a natural one to one correspondence between the direct summands of R and those of the algebra R-DR. If K is an algebraic number field and D is suitably chosen, then it is possible to study arithmetical properties of A by considering the decomposition of R. In particular if A is the group algebra of a finite group G, then known arithmetical properties of the characters of G can be extended in this manner. (Received September 6, 1956.)

5. I. N. Herstein: *A theorem on the characteristic roots of symmetric matrices.*

The following theorem is proved: let A be a real symmetric matrix and let $r_1(A)\geqq r_2(A)\geqq\cdots\geqq r_n(A)$ be its characteristic roots; then each $r_i(A)$ is a monotonically increasing function of each diagonal element of A, that is $\partial r_i(A)/\partial a_{jj}\geqq 0$. If B is the Jacobian matrix of the $r(A)$'s with respect to the diagonal elements of A then B is a stochastic matrix. (Received September 7, 1956.)

6*t*. M. A. Hyman: *Eigenvalues and eigenvectors of real nonsymmetric matrices.* Preliminary report.

By $(N-1)(N-2)/2$ real rotations, the real nonsymmetric matrix A of order N is transformed into matrix B with all zeros above the first super-diagonal: $B=R^{-1}AR$. The eigenvalue problem for B is then $(B-\lambda I)x=0$ where x is a column vector with

components $x_1, x_2, \cdots, x_N$. Setting $x_1=1$ and guessing λ, one calculates $x_2, x_3, \cdots, x_N$ and $F(\lambda)$ recursively; $F(\lambda)$ is the characteristic polynomial, evaluated for the trial λ. By successive approximation (see abstract below) a sequence of λ's (in general complex) is obtained which approaches an eigenvalue $\bar{\lambda}$ of $B: F(\bar{\lambda})=0$. Simultaneously, the vectors x approach the eigenvector $\bar{x}$. The corresponding eigenvalue and eigenvector of A are $\bar{\lambda}$, $R\bar{x}$. Using $\bar{x}$, B, one easily obtains matrix C, in form like B but of degree $N-1$ which has the same eigenvalues as A (less $\bar{\lambda}$ once). He repeats the process above to get an eigenvalue and eigenvector of C, then obtains matrix D of order $N-2$, etc. At any stage, the corresponding eigenvector of A is obtained by a single matrix times vector multiplication. If one has some information about the location of the eigenvalues of A, he can avoid the reduction process sketched above in whole or in part; this will save time and increase somewhat the accuracy of the results. (Received September 10, 1956.)

7*t*. M. A. Hyman: *Finding complex roots by interpolation.* Preliminary report.

Consider $F(\lambda)$, a general function of the complex variable λ. It is not assumed that $F(\lambda)$ is known explicitly, but that, given λ, $F(\lambda)$ can be found from a rule (for example, built up by recursion). Assuming the existence of a root of $F(\lambda)$ in a certain neighborhood, one obtains from three approximations to the root, $\lambda_1, \lambda_2, \lambda_3$, the approximation λ_4; λ_4 is a particular root of the quadratic function passing through $\lambda_1, \lambda_2, \lambda_3$. From λ_4 and the "best two" (in a certain sense) of $\lambda_1, \lambda_2, \lambda_3$, one forms λ_5. Continuing, a sequence $\{\lambda_i\}$ is constructed which usually converges to the desired root $\bar{\lambda}$. The method is designed for use with automatic computing machines. It is perhaps slower than more sophisticated methods which exploit differentiability, algebraicity, etc. of $F(\lambda)$, but appears simpler to carry through and enables the machine to handle a wider class of problems without special additional programming. The method can be speeded up using "acceleration" techniques. It causes the computing machine to use "real arithmetic" when pursuing a real root, but shift automatically to "complex arithmetic" when the root is complex (thus saving computer time). This "interpolation" method can be applied using 2, 3, 4, $\cdots$ points at each step, but to date the 3-point method described here seems the most useful. (Received September 10, 1956.)

8. H. G. Jacob: *Coherence invariant mappings of symmetric transformations.*

L. K. Hua (Amer. J. Math. vol. 50 (1949) pp. 8–31) has characterized one-to-one coherence invariant mappings of the set of $n\times n$ symmetric matrices over a field. A generalization of Hua's results is obtained by considering a nonsingular hermitian scalar product (x, y) defined on a vector space $\mathfrak{X}$ of dimension $\geqq 3$ over a field of characteristic $\neq 2$. A linear transformation T on $\mathfrak{X}$ of finite rank may be written as $T=\sum_1^n x_i\otimes y_i$; i.e. $zT=\sum_1^n (z, x_i)y_i$. A transformation is called symmetric if it equals its adjoint relative to the hermitian form. Under these conditions any one-to-one coherence invariant mapping σ of the set of symmetric transformations of finite rank is of the form $T^\sigma=\sum_1^n x_i\overline{A}\otimes y_iA+S$, where A is a semi-linear transformation on $\mathfrak{X}$ with an associated automorphism which commutes with the automorphism associated with the hermitian form. $\overline{A}=cA$ where c is a scalar, and S is a fixed symmetric transformation. In the event that A has an adjoint $T^\sigma=\overline{A}^*TA+S$, where $\overline{A}^*$ is the adjoint of $\overline{A}$. (Received September 10, 1956.)

9*t*. J. A. Kalman: *Lattices with involution.*

I. Nondistributive (but possibly modular) lattices with involution, or "i-lattices," are classified with respect to six laws each of which, for distributive i-lattices, is equivalent to normality (cf. Bull. Amer. Math. Soc. Abstract 61-4-536). II. It is proved that for each given normal i-lattice L there exists one and to within i-isomorphism only one normal i-lattice with zero L_Ω [closed normal i-lattice L^C] having the property: there exists an i-isomorphism π of L into L_Ω $[L^C]$ such that every i-isomorphism τ of L into a normal i-lattice with zero M [closed normal i-lattice N] is of the form $\tau=\pi\rho$ for some i-isomorphism ρ of L_Ω $[L^C]$ into M $[N]$. (An "i-isomorphism" is a lattice-isomorphism which preserves the involution; a normal i-lattice L is "closed" if, in L_Ω (with involution $z\to z'$ and zero θ), $x\wedge y=\theta$ implies $x=z\vee\theta$ and $y=z'\vee\theta$ for some z). III. The i-lattice with four elements and two zeros is shown to be the only subdirectly irreducible non-normal distributive i-lattice. (Research supported by the University of New Zealand Research Fund.) (Received September 11, 1956.)

10*t*. Joachim Lambek: *Subgroups of the direct product of two groups.*

Given groups A, B, C, let R and S be subgroups of $A\times B$ and $B\times C$ respectively, their *relative product* $R\cdot S$ is a subgroup of $A\times C$. Let R^- denote the *converse* of R, then $R\cdot R^-\cdot R=R$. Hence $R\cdot R^-$ and $R^-\cdot R$ are congruence relations on subgroups of A and B respectively, and R induces an isomorphism between the corresponding subfactor groups. This fact appears to have been discovered by Goursat. In particular, if R and S are congruence relations on subgroups of a given group, then $R\cdot S$ induces the isomorphism of the well-known Zassenhaus lemma. These results hold not only for groups, but for all algebras with a ternary operation $f(x, y, z)$ such that $f(x, y, y)=x$ and $f(y, y, z)=z$. Examples are loops with $f(x, y, z)=(x/y)\cdot z$ and quasigroups with $f(x, y, z)=(x\cdot(y\backslash y))/(z\backslash y)$ [see A. I. Mal'cev, Mat. Sbornik N.S. vol. 35 (1954) pp. 3–20]. To obtain Goldie's generalization of the Zassenhaus lemma [Proc. London Math. Soc. vol. 52 (1950) pp. 107–131], one must replace $R\cdot S$ by its "difunctional closure" due to J. Riguet [Comptes Rendus vol. 230 (1950) pp. 1999–2000]. (Received September 10, 1956.)

11. Marvin Marcus: *Convex functions of quadratic forms.*

Let $a=(a_1\cdots, a_n)'$ be a vector in the real n-space, E_n, and define the convex set $M(a)$ as the intersection of the half-spaces (I) $\sum_{j=1}^{k} t_{i_j}\leqq\sum_{j=1}^{k} a_j$, $1\leqq k\leqq n-1$, $1\leqq i_1<\cdots<i_k\leqq n$ and the hyperplane (II) $\sum_{j=1}^{n} t_j=\sum_{j=1}^{n} a_j$. Let H be the convex hull of the points Pa as P ranges over all n-square permutation matrices. Results (i) *If f is convex on H then* $\max_{t\in M(a)} f(t)\leqq\max_P f(Pa)$. (ii) *If f is convex on E_n and nondecreasing in each variable and $N(a)$ is the convex intersection of the half-spaces* (I) *then* $\max_{t\in N(a)} f(t)\leqq\max_P f(Pa)$. (iii) *If A is n-square Hermitian with eigenvalues $\lambda_1\geqq\cdots\geqq\lambda_n$ and $f(t_1, \cdots, t_k)$ is convex on the hypercube C defined by $\lambda_n\leqq t_j\leqq\lambda_1$, $k\leqq n$; and $\partial f/\partial t_j=0$ holds for at most one value of $t_j\in(\lambda n, \lambda_1)$ for each fixed $t_1, \cdots, t_{j-1}$, $t_{j+1}, \cdots, t_k$ then* $\max_{(x_i, x_j)=\delta_{ij}} f[(Ax_1, x_1), \cdots, (Ax_k, x_k)]=\max_{1\leqq i_j\leqq n} f(\lambda i_1, \cdots, \lambda i_k)$ *and a maximizing set of vectors $x_1, \cdots, x_k$ spans a k-dimensional invariant subspace of A.* For $f=\sum_{j=1}^{k} c_j t_j$, $c_j\neq 0$, (iii) is pertinent to a question raised by H. Wielandt (Proc. Amer. Math. Soc. vol. 6 (1955) p. 109). For $f=\sum_{j=1}^{n}\omega(t_j)$, (i) and (ii) follow from an inequality due to G. Polya (Proc. Nat. Acad. Sci. U.S.A. vol. 36 (1950) pp. 49–51). (Received September 7, 1956.)