BOOK REVIEWS

Propagation des ondes dans les milieux periodiques. By Léon Brillouin and Maurice Parodi. Paris, Masson et Cie., 1956. 348 pp. 4600 francs.

Throughout the development of theoretical physics one finds the periodic structure which distinguishes itself by the fact that common mathematical methods are available for what appears to be a host of different problems in physics. In the volume under review, Brillouin and Parodi have revised the earlier text of Brillouin (L. Brillouin, *Wave propagation in periodic structures*, New York, McGraw-Hill, 1946) and they give us an interesting account of periodic structures as they arose in Newton's time to those which arise in present day physics and technology.

The simplest periodic structure arose in the study of the motion of a uniformly loaded string (neglecting the mass of the string). The difference-differential equation, describing the dynamical behavior of the individual masses, occurs in many different physical problems, such as the uniform electric filter and the periodic loaded wave-guide. For finite structures, that is a finite number of masses in the case of the loaded string, we have the advantage that we can study the motion of the particles by what amounts to "finite Fourier series" since there are only a finite number of degrees of freedom in the system and this simplifies the methods of solution considerably. The book at hand is devoted to bringing to the attention of the reader the wide variety of physical problems which fall in this general category as well as a study of the mathematical methods. The mathematical methods are applied to systems with a finite as well as an infinite number of degrees of freedom and also to systems in two and three dimensions.

ALBERT E. HEINS

Structure of Rings. By Nathan Jacobson. American Mathematical Society Colloquium Publications, vol. 37, 1956. 7+263 pp. $7.70.

In the last dozen years a great deal of progress has been made in establishing a structure theory for associative noncommutative rings without chain conditions. A large share of the basic results were obtained by the present author and he is therefore eminently fitted to present the theory in its present form to the mathematical public. He has done so very successfully in the volume under review and has given us an essentially self-contained account which should be easily accessible to a reader with only the very basic notions of modern algebra at his disposal. Professor Jacobson has also shown very ably how the general theory specializes down to the case of rings with minimum condition. In fact, the book contains essentially all the results on semisimple rings with minimum condition to be found in, say, Artin, Nesbitt and Thrall, obtained as corollaries of more general theorems with, of course, the usual gain in insight.

The first two chapters contain a detailed presentation of the basic structure theory. The radical of a ring is defined as the intersection of the kernels of all the irreducible representations and all the usual equivalent characterizations are given. A ring A is semisimple if its radical is 0 and it is shown that A is a subdirect sum of primitive rings, i.e. rings possessing faithful irreducible right modules. Primitive rings are next treated via the Chevalley-Jacobson density theorem. This is first proved as a purely algebraic theorem in a formulation which is slightly more general than that of Artin, and then, after a careful formulation of the finite topology in a set of mappings, the topological version is given.

Chapter 3 applies these results to obtain the Wedderburn-Artin structure theory for rings with minimum condition. Actually, a somewhat more general class of rings is treated: it is only assumed that the ring modulo its radical satisfies the minimum condition, while the radical is only supposed to be S.B.I. in the sense of Kaplansky, a supposition fulfilled for radicals which are nil ideals.

The next three chapters deal with deeper properties of various classes of primitive rings. Chapter 4 concerns primitive rings with minimal one-sided ideals; Chapter 5 treats the structure of tensor products of primitive algebras, and in Chapter 6 a Galois theory for the ring of all linear transformations is developed.

Chapter 4 opens with the usual material on completely reducible modules which quickly yields the theory of the socle of a ring. Dual vector spaces are then introduced both algebraically and topologically and the density theorem of Chapter 2 is used to good effect in obtaining their basic properties. Next comes the fundamental Dieudonné-Jacobson structure theorem for primitive rings with minimal ideals which shows that such rings are subrings of the ring of all continuous linear transformations on a pair of dual spaces containing the socle of the latter ring. In Chapter 5, after developing the fundamental theory of tensor products (Kronecker products) of modules, a large number of structure theorems are proven. A typical one is the following due to Azumaya and Nakayama: Let A_1, A_2 be primitive algebras with faithful irreducible modules M_1, M_2 and commuting division rings D_1, D_2. Then the lattice of $A_1 \otimes A_2$ submodules of $M_1 \otimes M_2$ is isomorphic to the lattice of right ideals of $D_1 \otimes D_2$. Chapter 6 opens by developing the two basic tools of the Galois theory: A theorem of the Jacobson-Bourbaki type for rings of endomorphisms of completely reducible modules and the independence relations of automorphisms for the ring of all linear transformations. The Galois theory proper is then given and most of the known theorems concerning a simple ring with minimum condition and a simple finite dimensional subalgebra are derived from it.

A large part of the results of the earlier chapter have reduced down to questions concerning division rings. Accordingly, in Chapter 7 the author lays a foundation for a theory of division rings which are possibly infinite over their centres. The finite Galois theory due to Cartan and the author is developed essentially without using the more general theory of Chapter 6. Next the author presents an infinite Galois theory extending Krull's theory for fields. However, all the automorphisms in this theory must be outer, so that the finite theory is not covered by this extension. The algebraic theory of finite dimensional division algebras is next presented and the various extensions of Wedderburn's theorem about the nonexistence of finite division rings up to and including Nakayama's is given. Finally, there are several very welcome examples of division rings infinite dimensional over their centres.

The next chapter treats the upper and lower nil radicals of Baer as well as the locally nilpotent radical of Levitzki. There is a very neat proof of Levitzki's theorem to the effect that the maximal condition on right ideals makes every nil right ideal nilpotent. Weakly closed nil subsystems of a ring with minimum condition are also shown to be nilpotent.

Chapter 9 is devoted to a brief survey of the theory of the structure space of a ring. This is then used to obtain some of the Arens-Kaplansky representations of a ring as the ring of all continuous functions on certain topological spaces generalizing Stone's theorem about the representation of Boolean algebras.

The final chapter deals with three applications of the structure theory developed in the first nine chapters: (A) Commutativity questions; (B) Algebras satisfying a polynomial identity; (C) Algebraic algebras. The following theorem of Herstein is a typical sample of the results of (A): If $x^{n(x)}-x$ is in the centre, the ring is commutative. (B) contains a full account of the theory as it stands at present with several applications to structure theory. (C) is mainly devoted to giving the results that have been obtained so far on Kurosch's problem: "Is every algebraic algebra A locally finite?" Kaplansky's partial solution which asserts that the answer is yes if, (i) each primitive image of A satisfies a polynomial identity and, (ii) the radical of each homomorph of A satisfies a polynomial identity, is given. The last section presents Amitsur's recent results on algebraic algebras over nondenumerable fields. For example, it is shown that a full matrix algebra over such an algebra is again algebraic.

The book also contains a large number of pertinent examples and at appropriate places challenging problems that remain to be solved are raised. The proofs are clear, well motivated, and often an improvement over those found in the literature. Moreover, there are a great many very pleasant things in this work. These range all the way from recording hitherto unpublished, more or less well known facts in a definitive form to interesting new theorems. The following few examples are typical: (1) Very careful attention is paid to deriving algebra theorems from ring theorems; (2) If A is ring with unit the ideals of A_n are all of the form I_n, I an ideal in A; (3) The derivations of a primitive ring with minimal ideals into itself are completely determined. Indeed, let T be a continuous endomorphism of the underlying vector space such that for each scalar s, $sT-Ts$ is again a scalar, then every derivation is of the form $a'=aT-Ta$; (4) As part of the infinite Galois theory for division rings there is the following theorem: Let D be a division ring, H a group of automorphisms of D, and C the subring of H invariants. Suppose further that H contains every inner automorphism of D leaving C elementwise fixed. Then, if E is a division subring of D with $[E: C]_L < \infty$, every isomorphism of E into D can be realized by an element of H.

There is, however, one criticism that this reviewer would like to express: at several places in the book it is hard for the nonexpert to realize that the results in the text are not the best ones known. Of course, it is not possible for the author to prove all the results he states in their most general form, but the usefulness of the book could have been heightened by fuller references in several spots: (1) In Chapter 3, it is shown that the faithful irreducible modules for primitive rings with minimal ideals are unique, and a footnote points out that this is no longer true if either the hypothesis of faithfulness or the existence of minimal ideals be dropped. However, no examples are constructed and no references where such examples can be found are given. Indeed, the bibliography does not contain all the papers relevant to this point; (2) The commutativity theorem of Herstein given in Chapter 10 has been very extensively generalized by Nakayama in a paper in the 1955 Hamburger Abhandlungen. Although the paper does appear in the bibliography it is not referred to in the chapter, nor is the result in it mentioned; (3) In a paper in the 1954 Transactions of the American Mathematical Society Levitzki solved Kurosch's problem for a wider class of algebras than those mentioned in the text. Although this paper appears in the references to Chapter 10 the theorem in question is not cited.

But this, of course, is not a very serious drawback and the author is to be congratulated on having written this useful and encyclopaedic volume which, this reviewer feels, will be one of the standard works on the subject for some time to come.

ALEX ROSENBERG

Zetafunktionen und L-Funktionen zu einem arithmetischen Funktionenkörper vom Fermatschen Typus. By H. Hasse. Berlin, Akademie-Verlag, 1955. Abhandlungen der deutschen Akademie der Wissenschaften zu Berlin, Klasse für Mathematik und allgemeine Naturwissenschaften, Jahrgang 1954, Heft 4. 70 pp. DM 8.00.

In this memoir, the author has translated into his own language the reviewer's paper *On Jacobi sums as Grössencharaktere* (Trans. Amer. Math. Soc. vol. 73 (1952) pp. 487–495). Some minor points are given a fuller treatment, e.g. the question of determining the exact conductor of the characters which appear in the solution of the main problem (no general answer being given for that question). By restricting himself to the curve $X^m+Y^n=1$ (instead of the more general $aX^m+bY^n=1$), Hasse has been able to give a definition of the zeta-function, based solely on the reduction modulo prime ideals, which does not make the distinction between "ordinary" and "exceptional" primes. This case, however, is undoubtedly far too special to bring out the characteristic features of this important problem, on which one may consult more profitably the recent work of Deuring (Gött. Nachr., 1956) on elliptic curves with complex multiplication. One may also observe that, so far as the "ordinary" primes are concerned, such elliptic curves, as well as the abelian varieties arising from the decomposition of the Jacobian variety of a curve of the Fermat type, are all special cases of the abelian varieties with complex multiplication, whose zeta-function has now been determined by Taniyama (Tokyo Symposium on Number-Theory, 1955); in this sense the reviewer's treatment of the Fermat curve, as reproduced in substance in the present monograph, may be said to have no more than a retrospective interest.

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