An introduction to algebraic topology. By Andrew H. Wallace. New York, Pergamon Press, 1957. 7+198 pp. $6.50.

This book is an introduction to basic general topology, the fundamental group, and the homology groups. Aside from the choice of material (which is excellent), and the care devoted to the logical development (which is mostly very good), the outstanding feature is the extraordinary amount of motivation supplied. In this respect the treatment of the following topics is especially notable: continuity and neighborhoods, open sets and interior points, topology, induced on a subspace, Hausdorff space, frontier of a set, topological properties, compactness, connectedness, homotopy and homology, barycentric subdivision, excision. Furthermore the author has supplied mental images to an extent that is rare nowadays (for example, see pp. 81–82). The style is unfashionably leisurely; this is just right for a beginner, but might induce impatience in more advanced readers. The numerous exercises explicate and supplement the text, and are mostly not very challenging.

The homology theory that is presented is the singular theory (Chapters V–VIII), and calculation of the homology groups of simplicial complexes is deferred to Chapter IX in order to make use of the results of Chapters VI–VIII on induced homomorphism, barycentric subdivision, excision and homology sequence. This is in accordance with modern theory and is probably the most satisfactory method, although it has the unfortunate result that the apprentice topologist must wait patiently from p. 112 till p. 193 (the last page) to find out how to calculate any but the most trivial homology groups. (The last exercise in the book is to calculate the 1-st homology group of the torus.)

A similar situation exists with respect to the fundamental group. It is defined on p. 83, but the proof that the fundamental group of a circle is infinite cyclic is not completed till p. 161 (and consequently no nontrivial fundamental group can be calculated till then). The reason for this is that homology theory is used in the proof that the author uses. The reviewer prefers to make the proof of this important theorem independent of homology theory; this is usually done by referring to the theory of covering spaces, a theory which the author might well have developed, but can also be done quite simply by using the exponential map without explicit reference to covering space theory.

In defining the fundamental group, multiplication is defined only for closed paths (p. 75). The reviewer prefers to introduce the fundamental groupoid in order to have multiplication of paths available even when the paths are not closed. Although the author essentially does this in the exercises, he fails to make any use of it; for example he misses the possibility of shortening the proof of Theorem 21.

In the section on algebraic prerequisites, pp. 3–6, the author has been surprisingly careless. First of all his discussion of free groups, generators and relations is right out of the nineteenth century. That is, a free group is defined to be one that has no relations, and a relation is defined to be a product of generators and their inverses that is equal to 1; this completely ignores the fact that there is no domain where such a product makes sense until the free group has been introduced. Of course it is true that this "cart before the horse" procedure will bother only the more perceptive students, and it *is* much shorter than a correct explanation, but the author should at least have indicated to the reader that this is a slightly bowdlerized version. Secondly he defines finite generation, cosets and quotient groups only for abelian groups. It is difficult to see why this was done, since no space is saved thereby. Furthermore it is misleading by implication, and would seem to make it more or less impossible to do anything *with* the fundamental group (which is only rarely abelian). For example it is shown on pp. 160–161 that $\pi(E)$ is mapped onto $H_1(E)$ with the commutator subgroup as kernel; apparently the reader is not to be allowed to conclude that $H_1(E)$ is isomorphic to the commutator quotient group of $\pi(E)$. Finally, the statement on p. 4, l. 8*-5* is false, as was shown by the reviewer in Annals of Mathematics vol. 49 (1948) pp. 497–498.

Although the notation has been generally well thought out, the author has been so unfortunate as to succumb to the current fad of writing $g \circ f$ for the composition gf and to the monstrosity

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for the inverse of f. On p. 75 the product of two (closed) paths, f and g, is defined and denoted fg. It would have been better to have written this $f \circ g$ or $f \cdot g$ to avoid confusion with composition (and this confusion would be very real if the paths under consideration were paths in the unit interval I).

In spite of these minor defects this would be an excellent book for a course for advanced undergraduates, or even for beginning graduate students. It is especially suitable for independent reading; there is no better book to put into the hands of a student who wants to start learning topology for himself.

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