(Perturbation theory), the coefficient function $q(x)$ is replaced by $q(x)+\epsilon s(x)$, where $q(x)\to\infty$ as $|x|\to\infty$, $s(x)\geqq 0$ and $\epsilon\geqq 0$. In Chapter XX (Perturbation theory involving continuous spectra), it is supposed that $q(x)\to\infty$, $s(x)\to-\infty$, $q(x)=o(|s(x)|)$ as $|x|\to\infty$. The main problem is an estimate of the contribution to the expansion of f, in the perturbed case, of the λ-values outside a small open set containing the discrete eigenvalues of the unperturbed problem. Chapter XXI (The case in which $q(x)$ is periodic) is devoted mainly to the 1-dimensional case. Finally, there is Chapter XXII (Miscellaneous theorems) mentioned above.

In a number of chapters, the general theory is illustrated by applications to interesting examples.

PHILIP HARTMAN

Linear operators. Part I: *General theory.* By Nelson Dunford and Jacob T. Schwartz, with the assistance of William G. Bade and Robert G. Bartle. New York, Interscience Publishers, 1958. 14+858 pp. $23.00.

In many ways this is an impressive book. The first way in which it is certain to make an impression on anyone who picks it up is by sheer size; an approximate word count reveals that it is only a little longer than *Doctor Zhivago.* Remember, however, that this is only the first volume, containing eight out of a total of twenty chapters. The work is intended to constitute an organic unit; the reasons for binding it in separate volumes are more practical than mathematical. Since at the time that this report is being written the second volume has not yet appeared, what follows refers to the first volume only.

The book makes use of several expository devices, which, while they are not new in concept, are here applied with such an astonishing degree of completeness and on such a gargantuan scale as to deserve special mention. On the end papers, for instance, there is a graph of the interdependence relations among the sections (of the first volume only) that is the most complicated thing of its kind the reviewer has ever seen. The graph is not embeddable into the plane, and it is not at all a trivial task to decipher the information it contains.

A helpful device is a black marginal arrow marking the theorems that may look insignificant but in fact play an important role in later developments.

At the end of most chapters there is a section of notes and remarks. These sections almost completely replace footnotes (a splendid idea). The remarks are not mere afterthoughts and references; they are scholarly asides (sometimes a little on the pedantic side) such as a specialist lecturing on the subject might insert to enliven and broaden the discussion. Altogether the notes and remarks take up a little more than 100 pages of the book.

Every chapter is liberally provided with exercises into which the authors managed to cram many extensive theories. Examples: the Jordan canonical form for complex matrices, and the Weierstrass polynomial approximation theorem in n-dimensional space. Altogether the exercises cover about 100 pages also.

The various lists at the end of the book (references and indices) are of a size that is appropriate for an encyclopedic work of this kind (altogether well over 100 pages). The authors apparently made a serious effort to give a complete list of references. Included are a paper by Abel (1826) and a recent one by Zygmund and Calderón (1956); there is a very good coverage of the Russian literature.

Here is a survey of the contents. Chapter I, *Preliminary concepts.* This is a quick treatment of basic set theory, topology, and algebra. The set theory is the kind that is technically known as "naive." The axiom of choice is not stated; Zorn's lemma is proved. The definition of Cartesian product occurs in an exercise. The treatment of topology begins with the definition of a topological space and goes through the Tietze extension theorem, the Baire category theorem, and the Tychonoff theorem on products of compact spaces. The treatment of algebra includes the definitions of groups, vector spaces, algebras, and determinants. Most of the facts are on the elementary level of universal algebra, except that, surprisingly, a proof of Stone's theorem on the representation of Boolean algebras is included. Chapter II, *Three basic principles of linear analysis.* They are the principle of uniform boundedness, the interior mapping principle, and the Hahn-Banach theorem. Chapter III, *Integration and set functions.* The theory is presented so as to include vector-valued functions and/or measures, and to do so with the minimum of fuss. The material is standard: it includes the Lebesgue dominated convergence theorem, the Radon-Nikodym theorem, and the theory of product measures. There is a treatment of vector-valued analytic functions of a complex variable.

Chapter IV, *Special spaces.* This is the longest chapter. Its purpose is to discuss the standard problems (representation of linear functionals, reflexivity, compactness, etc.) in the standard spaces (L_p, C, BV, etc.). The discussion is detailed and thorough; it includes facts that have hitherto been scattered all over the literature. There is a tremendous amount of material here. Examples: the Riesz-Markoff theorem and the Stone-Čech compactification. There is a table (stretching over six pages) that summarizes the main results. Chapter V, *Convex sets and weak topologies.* The main topics are: separation theorems for convex sets, the Tychonoff-Alaoglu theorem, Eberlein's theorem on sequential compactness, the Krein-Milman theorem, and the Schauder fixed point theorem. For locally convex spaces, only what might be called the "classical" theory is given—pre-Bourbaki and pre-Grothendieck.

Chapter VI, *Operators and their adjoints.* Main topics: completely continuous operators, and the Riesz-Thorin convexity theorem. There is a tabular presentation of the principal representation theorems for operators from and to the standard spaces. Chapter VII, *General spectral theory.* The discussion begins with finite-dimensional spaces, includes the Riesz theory of completely continuous operators, and includes also an introduction to perturbation theory. Chapter VIII, *Applications.* The applications concern semigroups (the Hille-Yosida theorem), and ergodic theory (mean and individual).

The terminology and the notation are almost always standard and easy to assimilate. The exposition is never watery. Things proceed at a good clip; the definitions and the proofs are concise and neatly formulated. A tremendous enterprise such as the authors have undertaken is more likely to fail than to succeed, and existence itself is more than half the proof of success. The authors deserve thanks for their labors and congratulations on their achievement.

PAUL R. HALMOS

Introduction to logic. By Patrick Suppes. Princeton and New York, Van Nostrand, 1957. 18+312 pp. $5.50.

One can distinguish at least three attitudes towards the increasingly important role of logic in the undergraduate mathematics curriculum; the *reactionary* attitude which denies it any place; the *moderate* attitude which regards it as a "luxury" subject, to be made available to those advanced students who are especially interested; and the *progressive* attitude which regards it as one of the earliest and most basic skills which a major should learn.

Rosenbloom's *Elements* is excellent for the purposes of the moderates; but until the appearance of Suppes' book the progressives had the alternatives of teaching from notes or bowdlerizing one of the existing texts. The better of those texts shared the feature that they became embroiled in logical questions for their own sake rather than as a tool of mathematics; some of them (notably Fitch's and Rosser's) fell on this account awkwardly between being an undergraduate text