

and not many illustrative examples. Thus, in §2.15 only seven explicit integrals which can be found using residues are mentioned. The discussion of the last on page 119 with integrand  $(\ln z)^2/(1+z^2)$  is inadequate, since if  $I_1$  is the integral from 0 to  $\infty$  and  $I_2$  is that from  $-\infty$  to 0, some nontrivial manipulation is needed to show that  $I_2 = I_1 - \pi^3/2$ , a preliminary to  $2I_1 - \pi^3/2 = -\pi^3/4$ , which then gives the result correctly stated at the end of 2.153.

The omission of examples together with the brevity of the exposition, explain how the author has managed to include so many topics in a single volume. But on the whole, this book may be highly recommended to any reader desiring a broad knowledge of function theory from a compact presentation.

PHILIP FRANKLIN

*Contributions to the theory of games*, Vol. 4. Ed. by A. W. Tucker and R. D. Luce. (Annals of Mathematics Studies, No. 40.) Princeton University Press, 1959. 9+453 pp., \$6.00 (paperbound).

This is the fourth, and at present writing intended to be the last, volume on the theory of games in the present series. It is devoted to  $n$ -person games, which are very much more complicated than two-person games because formation and disruption of coalitions and making side payments are permitted. The present volume, as its predecessors, is prefaced by an introduction which describes the general state of the theory and summarizes the contents of each paper. This introduction is itself an excellent review and makes further comment here unnecessary. The general remarks made by the reviewer in his review of the preceding volume of this series (Bull. Amer. Math. Soc. vol. 65 (1959) pp. 101–102) apply here as well.

J. WOLFOWITZ

*Lie groups*. By P. M. Cohn. (Cambridge Tracts in Mathematics and Mathematical Physics, No. 46.) New York, Cambridge University Press, 1957. 7+164 pp., \$4.00.

It is only in recent years that the study of Lie groups as global objects has come to be considered a subject of general interest. Until that time, it was customary to study only the "group germ" of a group which, upon analysis of the best-known literature, was some unspecified neighborhood of the identity element of the group. Among the familiar works along this line we find Eisenhart's book on continuous groups, while a quite recent discussion of the same nature is found in a chapter of Schouten's *Ricci calculus* (second edition).

At quite an early date, Elie Cartan was aware of the global nature of Lie groups; of connectedness and compactness and such; but his work never found a really large audience. This was partly due to his incomplete proofs, but mainly to the difficulty of understanding his general terminology and machinery.

The one text that strongly promoted the interest in Lie groups as global objects was Chevalley's well-known book in the Princeton series. Not only did Chevalley give a coherent and logically complete description of the general nature of Lie groups; he also gave it in a language which could be understood much more readily than Cartan's. In fact, this one book has done more than that: it has stimulated great interest in an "intrinsic" (=coordinate-free) approach to differential geometry; and it is still frequently quoted in research publications as a general reference for the methods.

Thus, Chevalley's text achieved three things at once: (i) it treated Lie groups as global objects; (ii) it was rigorous; (iii) it had a "new language." While the first two items are generally approved of as positive gains, the third is of a more controversial nature. The modern differential geometer is very happy about it—by definition!—the topologist and algebraist also prefer mappings, linear functionals and such; but the classical differential geometer and the (much more numerous) analyst may very well find that they get too much of too many different kinds of formalism at once. After all, aren't Lie groups essentially just groups whose elements can be described by a number of parameters, while the group operations are expressed by analytic functions of these parameters? In Chevalley, he frequently cannot find the parameters (co-ordinates) or the functions. For somewhat similar reasons, a student with an adequate knowledge of analysis, but little mathematical sophistication may easily find himself snowed under.

It seems that Cohn wrote his little booklet, here under review, with just these latter groups of people in mind. He has made a serious and quite successful effort to preserve the full advantage of the first two objectives of Chevalley: global treatment and rigor; but he has "softened up" on the language of presentation. While any specialized language with clever possibilities—whether "classical" or "intrinsic"—can be mastered only after considerable practice, the author has tried to do away with anything that cannot be considered a straightforward expression in truly simple terminology. As a consequence, few results "come tumbling out" (which is only possible after creating sophisticated tools in some previous chapter), but proofs are of reasonable length and in each instance the reader sees quite clearly what is going on.

As a result, we have here a book which easily lends itself to a course (it will particularly appeal to a nonspecialist) or better even, a seminar in which inexperienced students can actively participate.

Lie groups are infinite groups which are topologized (that is, concepts like "open set" and "neighborhood" are meaningful), and in which every element has a neighborhood whose elements which can be "described" (co-ordinatized) by a certain number of parameters. Elements that lie in overlapping neighborhoods have more than one description (set of co-ordinates); and any two of them are assumed to be related by analytic functions. A very simple case is the group of  $n \times n$  nonsingular real matrices, where  $n^2$  numbers (the entries of the matrices) describe all elements of the group at once. In more complicated groups (such as the rotation groups in 2 or 3 dimensions) such a simple description by one set of co-ordinates is no longer possible; the groups can only be covered by a number of "patches," in each of which one can set up a co-ordinate system.

An essential part of the definition of a Lie group is the fact that the group operations (product, inverse) can be expressed by analytic functions in terms of the local co-ordinates. Thus, the differentiable structure (manifold structure) and the group structure are intimately interwoven. This fact finds its reflection in the exceedingly pleasant happenings that go on in the tangent spaces to the group manifold, and it gives rise to vector fields and differential forms on the groups with so-called left- or right-invariance properties. Thus emerges a vector space with additional multiplicative structure derived from the group structure: the Lie algebra. The fact that it completely determines the nature of the group in a neighborhood of the identity element is—in a vague and general way—the content of Lie's three Fundamental Theorems.

The material loosely summarized above covers five chapters of the book, entitled Analytic Manifolds, Topological Groups and Lie Groups, the Lie Algebra of a Lie Group, the Algebra of Differential Forms, and Lie's Fundamental Theorems. After this he devotes one chapter to subgroups and homomorphisms, and the last one to the universal covering group. An appendix is devoted to integration of systems of partial differential equations. No attempt is made at any place to go into the classification of groups (simple, nilpotent, etc.); into representation theory, nor into the study of strictly topological invariants.

Besides generally commending the author for his presentation of several delicate points of a predominantly topological nature, such

as, for example, closed subgroups, special mention should be made of the discussion of local groups and group germs. He clearly shows that the classical approach—even when restricted to a neighborhood of the identity—was really inadequate.

ALBERT NIJENHUIS

*Introduction to algebraic geometry.* By Serge Lang. New York, Interscience, 1958. 11+260 pp., \$7.25.

This book, an introduction to the Weil-Zariski algebraic geometry, is an amplification of lecture notes for one of a series of courses, given by various people, going back to Zariski about a dozen years ago. Restricted to qualitative algebraic geometry (i.e. no intersection multiplicities, not even Chow coordinates), it is an admirable introduction to Weil's *Foundations* and, more generally, the whole of the modern literature as it existed before the advent of sheaves.

The text starts with the usual material on valuations, places, and integral dependence. From this are swiftly derived all the elementary facts on varieties (first affine, later projective or abstract, all either relative to a fixed ground field or to a universal domain) such as irreducibility, generic points, the Hilbert Nullstellensatz, dimension, dimension of intersections, products, projections, and correspondences. The effects of ground field extensions are given, and a subsidiary discussion of linear disjointness and separability leads to the notions of field of definition of a variety and rationality of a cycle over a field. It goes without saying that a large number of extremely important facts crop up almost incidentally (examples: separable points are dense in the Zariski topology, an abstract variety is a regular image of one in a projective space). Next come normal varieties, normalization of a variety (in a larger field), Zariski's original proof of his Main Theorem, divisors and linear systems (including the associated rational map and finite dimensionality on a complete variety), the last theorem of *Foundations* and the least field of rationality of a cycle. A brief chapter on derivations and differential forms (after Koizumi) is followed by one on (absolutely) simple points that includes a bit on local rings and the irreducibility of the generic hyperplane section. Two final chapters direct attention to a topic of current interest: connected algebraic groups are defined and some first results on these are proved (e.g. completeness implies commutativity, and in this case a map of a product variety decomposes), the Riemann-Roch theorem for curves is proved (following Weil and repartitions, for an algebraically closed ground field, but