THEOREM. *Every commutative Moufang loop which can be generated by n elements, where $n \geqq 3$, is centrally nilpotent of class at most $n-1$.*

This result in fact is so recent that it does not appear elsewhere in the literature.

On the whole the book is highly original. Many of the published results appear in more lucid form. The book is also very readable and appears free of misprints worth mentioning. In other words what it chooses to treat is done in an excellent way. Without a doubt this book is a must for anyone even vaguelv interested in the study of loops. To the beginner it offers a survey of the literature on binary systems coupled with an excellent bibliography. It is also a work that is likely to stimulate interest and further research in binary systems.

ERWIN KLEINFELD

Foundations of geometry, Euclidean and Bolyai-Lobachevskian geometry, projective geometry. By K. Borsuk and Wanda Szmielew. Revised English translation. Amsterdam, North-Holland Publishing Company, 1960. $12.00.

Euclid's long lasting popularity is readily explained by the lack of progress on the field covered by the *Elements* during more than twenty centuries although perhaps this sterility was merely a consequence of the authority imposed by the Greek geometrician. The same argument would not suffice to explain the unshaken confidence with which Hilbert's *Grundlagen der Geometrie* is still considered by many people as the definitive revelation of geometric truth. Too much has happened in the last sixty years in the axiomatics of geometry. When I had the opportunity to review the 8th edition of Hilbert's *Grundlagen der Geometrie* in Nieuw Archief voor Wiskunde, I tried to answer this question by a detailed analysis of the historical context of that work, and by an appraisal of its positive qualities as well as of its drawbacks. The still overpowering appeal of Hilbert's work is attested anew by the fact that two renowned Polish mathematicians have engaged in the difficult and not too grateful task of elaborating Hilbert's work and adapting it in detail to a more modern concept of mathematics. Though in Euclidean and Lobachevskian geometry Borsuk's and Szmielew's work covers only a small part of Hilbert's booklet, its extent is three or four times that of its predecessor. This proves anew, if ever proof was needed, that the brevity of Hilbert's work was bought by extensive, though mostly minor, omissions. It also proves that Hilbert's lay-out is too complicated and that it can hardly serve as a basis of a simple axiomatic introduction into geometry. Of course, it must be noticed that such was not Hilbert's aim. All details in the *Grundlagen der Geometrie* were subordinated to the logical analysis, especially to showing the independence of numerous axioms, and little care was bestowed on simplifying the structure of the system. It is a fact that little if any attention is paid by Borsuk and Szmielew to the essentially new features of Hilbert's work (as little as not having the opportunity to mention Pappus' theorem). Thus with respect to content, their book reflects the foundations of geometry as Hilbert encountered them when he started his work. Of course, formally, Hilbert's idea of axiomatic structure and implicit definition is fully dealt with, though obscured by the clumsy lay-out of Hilbert's system. The huge respect paid to Hilbert's work is also expressed in the formulation and the arrangement of the various axioms which are essentially Hilbert's. The early introduction of the axioms of order (based as in Hilbert's work on the less practicable relation of betweenness) provides the authors with a welcome opportunity to step over into topology and to give detailed proofs of fundamental theorems which are not even mentioned by Hilbert (though not of Jordan's theorem). By methods and results available in the current literature this chapter could still have been improved. In the theory of congruence the authors follow Hilbert in refraining from group theory notions and tools, in spite of the highly artificial formulations to which this abstention has led. Consequently much care had to be bestowed on proving existence and uniqueness theorems for congruent mappings of the whole plane. This treatment is particularly interesting, though it does not involve the use of transformation and group theory notions in the sequel. Thus, topics like the Klein model of hyperbolic geometry do not appear in a group theory context, but as examples *ad hoc.*

Though this attempt to elaborate Hilbert's program is highly interesting as much where the authors have closely followed Hilbert's footsteps as where they have deviated, a few instances of a strong dependence on Hilbert are difficult to explain. The most striking one is a historical curiosity like Hilbert's example of a non-Desarguesian geometry, which immediately after the publication of Hilbert's *Grundlagen* was superseded by much simpler examples, but which still appears in the present book. Apart from such rare instances the authors have achieved remarkable things within the narrow frame in which they have designed their work. Without doubt essential improvements of Hilbert's layout would have essentially modified the character of Hilbert's system. The present work is particularly remarkable as an example of how to adapt a work that looks old fashioned in linguistic expression and technical details to modern terminology and mathematical technic. In this respect, its educational value might be rather high.

A few historical remarks might still be added. The well-known Cayley-Klein model of hyperbolic geometry is called the Klein-Beltrami model by the authors. At several places they assert that Klein constructed it on the basis of the earlier ideas of Beltrami. This revival of outlived priority objections is contradicted by Klein's historical report. Klein acknowledges his dependence on Cayley which is also supported by internal evidence, and for similar reasons it is quite clear that he became acquainted with Beltrami's paper only afterwards. Perhaps Cayley's non-Euclidean metric has been overlooked by the authors because it leans on group theory, which is of secondary importance in their work. Beltrami discovered that under a suitable parametrization the geodesics of surfaces with constant negative curvature become straight lines. Of course, this fact is related to Cayley's metric. The connection, however, with the projective group, essential in Klein's construction, could not be found by Klein in Beltrami's paper, but only in Cayley's.

Another point of historical interest or rather of historical curiosity: As "Thales' theorem" the authors introduce a theorem on similar triangles. In the German textbooks Thales is responsible for the rectangularity of the triangle in the semi-circle. Mathematical folklore may list still other Thales' theorems in other countries. By Eudemos, Thales is credited with no theorem on similitude. Maybe some amusing story about Thales visiting Egypt and surveying pyramids inspired some textbook writer to call this theorem Thales'. I wonder whether there may not be a folklore in which the formula for the sum of an arithmetical progression is called Gauss' theorem.

HANS FREUDENTHAL

Introduction to functional analysis. By A. E. Taylor. New York, John Wiley and Sons, 1958. xvi+423 pp. $12.50.

This book provides a carefully organized, readable, and unusually complete introduction to functional analysis. The central theme is the theory of normed linear spaces and operators between normed linear spaces. Where the property of completeness is crucial, Banach spaces are used. Special developments are given for Hilbert space when results of a distinctive nature can be obtained. There are many problems. These are chosen carefully and make a significant contribution to the completeness of the book.

The first two chapters introduce needed vector-space and topologi-