RESEARCH PROBLEM

3. Albert Wilansky: An elementary inequality.

Given $\sum |b_k| < \infty$, must there exist a constant $M$ such that whenever $\{x_n\}$ is a convergent sequence satisfying $|\sum_{n=1}^{\infty} b_kx_k + x_{n-1} + x_n| < 1$ for all $n$, then $|\lim x_n| < M$?

Remarks. This asks, of course whether $\lim$ is continuous in a certain topology. The result is true if the term $x_{n-1}$ is omitted (Mazur, see [1]), it is also true if all $b_n = 0$ [since $\lim x_n = (1/2) \lim (x_{n-1} + x_n)$].

The given transform is $Bx + 2Ax$ where $A_n(x) = (1/2)(x_{n-1} + x_n)$, $B_n(x) = \sum_{k=1}^{n} b_kx_k$, $B$ is in the radical of the Banach Algebra of triangular conservative matrices, $A$ is regular. See [1; 3].

If the answer to the question is no, this will provide a second example (the first is due to Zeller, see [2]), of a coregular matrix with no equivalent regular matrix.

REFERENCES

2. A. Wilansky, Summability; the inset, the basis in summability space, Duke Math. J. vol. 19 (1952) especially p. 657.

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