this is perhaps an indispensible feature of a work which is intended
to lead the reader into consideration of the many open problems re­
mainin in this field.

A summary of the contents is as follows: I. A survey of classical
representation theory, ordinary and modular; II. Young tableaux
and the representations of the symmetric group; III. The connection
between the representations of the symmetric group and those of the
full linear group; IV. The calculation of characters corresponding to a
given Young tableau; V. Blocks of representations; condition for two
representations to belong to the same block; VI. Analysis of the set
of representations in a block and computation of the number of modu­
larly irreducible representations in a block; VII and VIII. Compu­
tation of the decomposition of an ordinary representation into modu­
larly irreducible components; new results due to J. H. Chung, O. E.
Taulbee, Diane Johnson and the author.

Note. The author has pointed out to me that in paragraph 8.2 ff.
repetitions of an admitted permutation may belong to different com­
ponents when \( p = 2 \). Consequently corrections are necessary to the
tables 2.7–2.10 and Theorem 8.41.

ANDREW H. WALLACE


The latest of Synge's books shares the virtues of his earlier ones: it is well written, it is interesting, it is different from other books on relativity.

Mathematicians and theoretical scientists can be roughly divided
into geometers and algorists, Riemann being a good example of a
geometer, even when he works in analysis, and Descartes an example
of an algorist, even when he works in geometry. Synge is a geometer,
and his two books on relativity give special pleasure to those who,
like the reviewer, think best when they can see pictures in their mind.
One can go further and say that only geometers can fully understand
Einstein's theory of relativity. Students of relativity who wish to be­
come geometers should read Weyl's Space-Time-Matter or Synge's
Relativity, and preferably both.

Relativity: The general theory is the continuation of Relativity: The special theory (North-Holland, 1956). The first three chapters cover
mathematics and the kinematics of general relativity. Chapter IV
reviews continuum mechanics and introduces the differential equa­
tions of the gravitational field. Chapter V discusses some solutions
and the Cauchy initial value problem for the gravitational field equa­
Chapter VI gives some isolated results connected with the difficult problem of integral conservation laws and extensive variables, such as energy, for a gravitating system. Chapters VII and VIII cover the classical solutions of Schwarzschild and of Weyl and Levi-Civita, and cosmological models, including the rotating Gödel universe. The last three chapters cover plane gravitational waves, electromagnetism and geometrical optics.

The book contains a mathematical novelty, the systematic use of Ruse's two point invariant. This "world-function" is, to within a trivial factor, the square of the geodesic distance between two events, regarded as a function of their eight coordinates. The world-function is applied to an interesting new approximation procedure which permits development in power series without abandoning the tensor calculus. It remains to be seen how useful the approximation method will prove in the exploration of new solutions and properties of Einstein's field equations.

Theoretical scientists were classified above into geometers and algorists. However, there is another independent classification which is equally relevant. Theoretical scientists can be roughly divided into mathematicians and physicists. The two differ in their concept of "truth." To the mathematician the great truth is consistency, the great sin is inconsistency. To the physicist logical consistency, as such, is no virtue. He is groping along a dark road towards a theory which will explain all of nature; he does not necessarily believe that the road is finite, but he does know that the ultimate theory at the end will automatically be consistent. He is, therefore, not overly concerned with inconsistency in a provisional theoretical system; inconsistency is merely one of many indications that he is still stumbling on the dark road and far from his destination. A theory which explains a large region of experimental and observational facts, even though it is inconsistent on the blurred boundary of the region, tells him that he is on the right road. It is this road which is the physicist's truth; to him the great sin is to be lured away from it towards bright and beautiful mirages on either side.

One can say that only physicists can fully understand Einstein's theory of relativity. Synge, a geometer rather than an algorist, is also a mathematician rather than a physicist. The reviewer's only criticism of the book can be traced directly to this and concerns sins of omission. Synge discusses the principle of equivalence only in the preface and only to dismiss it immediately as a mere historical curiosity. To the reviewer the principle of equivalence is the very heart of Einstein's theory of gravitation; it is the only part of general
relativity which is supported by large numbers of accurate experiments. Even within the theoretical framework of general relativity it can surely not be regarded as unimportant; for example, the principle of equivalence, when combined with symmetry properties and the fact that Newtonian gravitational theory must be a valid approximation, leads directly to the conclusion that the space-time of a massive spherical body at rest must be curved. The other important omission is the subject of variational principles for the gravitational field and other fields which interact with it, and the beautiful and deep relationship discovered by Hilbert and Klein between the invariance properties of the action integral and the existence of differential conservation laws. This theory of the structure of field theories is required if one attempts to understand the energy momentum tensor not as a mere phenomenological description of matter, but as a sum of contributions from other fields (electromagnetic field, electron field, etc.) which are the joint sources of the gravitational field. It is also required if one attempts to construct other gravitational theories of the type of general relativity, which share with it some desirable characteristics, covariance and the existence of conservation laws.

Synge’s books belong on the shelf of every serious student of relativity—but they should be flanked by other books on the subject. John Power and John Jameson can be proud to have Relativity: The general theory dedicated to them.

Alfred Schild

Mathematical foundations of quantum statistics. By A. Y. Khinchin
(Translation from 1951 Russian Edition edited by Irwin Shapiro).

Let \( \mathcal{H} \) be the Hilbert space whose unit vectors define the pure states for a quantum mechanical particle. Let \( H \) be the energy operator and suppose that this operator has a pure point spectrum with eigenvalues \( 0 \leq \epsilon_1 \leq \epsilon_2 \leq \cdots \). For each positive integer \( N \), let \( \mathcal{H}_N \) denote the \( N \) fold tensor product of \( \mathcal{H} \) with itself and let \( H_N \) denote the sum

\[
H \times I \times \cdots \times I + I \times H \times I \cdots \times I + \cdots
\]

\[
+ I \times I \times I \cdots H
\]

where \( I \) is the identity and each product has \( N \) factors. Then \( \mathcal{H}_N \) and \( H_N \) are the Hilbert space and energy operator for a system of \( N \) noninteracting replicas of the particle described by \( \mathcal{H} \) and \( H \). Let \( \mathcal{H}_N^S \) denote the closed subspace of \( \mathcal{H}_N \) consisting of all symmetric members of the tensor product and let \( \mathcal{H}_N^A \) denote the closed subspace of \( \mathcal{H}_N \) consisting of all anti-symmetric members of the tensor product.