RESEARCH PROBLEM


Let $M$ denote a linear manifold (i.e. translate of a linear subspace) in $n$-dimensional real space. For each real $p > 1$ there is a unique point $x_p = (x_{p1}, \ldots, x_{pn})$ on $M$ for which the norm $(\sum_{j=1}^{n} |x_{pj}|^p)^{1/p}$ is a minimum. Prove or disprove the conjecture that $\lim_{p \to \infty} x_p$ exists in all cases. The conjecture has been established by simple arguments when $n \leq 3$, when $M$ has dimension 1 or $n - 1$, and when there exists a unique point $x_0 = (x_{01}, \ldots, x_{0n})$ on $M$ for which $\max_j |x_{0j}|$ is a minimum. (Received January 20, 1962.)