
Write

\[
 f_N(x) = \max_{\{x_i\}} \sum_{i=1}^{N} g_i(x_i)
\]

where the maximum is taken over the region \(x_i \geq 0, \sum_{i=1}^{N} x_i = x\), with \(x > 0\). Under what conditions on the sequence \(\{g_i(x)\}\) can we assert that \(f_N(x) \sim N \phi(x)\) as \(N \to \infty\)?

Using the functional equation technique of dynamic programming, we see that

\[
 f_N(x) = \max_{0 \leq y \leq x} [g_N(y) + f_{N-1}(x - y)].
\]

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Consider a system of \(N\) simultaneous differential equations of the form \(dx_i/dt = g_i(x_1, x_2, \ldots, x_N), x_i(0) = c_i\), where the \(g_i\) are polynomials in the components \(x_i\) or, more generally entire functions.

Under what conditions on the \(g_i\) do there exist functions \(h_i(y_1, y_2, \ldots, y_k), k < N\), entire as functions of the \(y_i\), with the property that the functions of \(t\) defined by \(f_i = h_i(y_1, y_2, \ldots, y_k), i = 1, 2, \ldots, k\), satisfy a set of simultaneous differential equations of the form \(df_i/dt = G_i(f_1, f_2, \ldots, f_k)\), where the \(G_i\) are entire functions of their arguments? When these new variables exist, how does one determine them?

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