étrie des espaces de Riemann. Only in the closing notes do the authors unbend and approach showing the beginner what differential geometry is all about; on the other hand there is not enough advanced material to really serve the expert as a reference book.

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Introduction to knot theory. By R. H. Crowell and R. H. Fox. Ginn and Co., Boston, Mass., 1963. 10+182 pp. $8.00.

It takes a bit of hunting to find any reference to the real world in most modern books on topology. The roots of knot theory however are in our physical world, so that it is not surprising that this introduction to knot theory places itself in the position of attacking a "practical" problem. What is surprising is the degree of practicality maintained throughout the book, for it is frequently the case that a mathematician's solution to a *real* question is useless from a pragmatic point of view.

The mathematician who is unfamiliar with knot theory, but familiar with topology will find in this book an emphasis which may be foreign to him, that is, given a knot (say by a photograph) the absorption of the contents of this book makes relatively easy the actual computation of invariants of the knot.

The mathematician who is unfamiliar with topology will find this book an excellent starting point. The juxtaposition of a theory with its applications makes for interesting and instructive reading. It is often very hard to understand a theorem in vacuo, and this book is so well knit that this unfortunate state of affairs is generally avoided.

It must be said that although the foregoing comments may seem to imply a rather special and simpleminded sort of mathematics being done in this work, such is very definitely not the case. Although the distinguishing of knots sounds innocent enough, the mathematics used is occasionally quite general, occasionally quite sophisticated, occasionally quite interesting, and always precise. That is not to say that the book is of a formal character; it is formal only when it needs to be, formality for its own sake is carefully avoided, a virtue which is unfortunately not universally agreed upon as such.

Reluctance to buckle down and *learn* the theory put forward in this book will not prevent the reader from profiting from a reading. There is often a good deal of informal discussion prior to making a definition or a proof, and this makes it possible to learn about the subject without wading through yards of notation. On the other hand this aspect of the presentation also makes it possible to better understand the details of the subject.

Let us take a cursory look at the organization and actual content of the book.

Knot theory utilizes algebra to a high degree, and in Chapters 4 and 7 are found some of the fundamental algebraic concepts which are used.

Chapter 4 begins with a historical foundation for the idea of a group presentation which is the chief topic of this chapter. The (homotopy) type of a presentation being defined, the authors proceed to define mappings of a presentation. This is followed by a careful proof of the theorem of Tietze. (Any two finite presentations of the same group are related by a finite sequence of elementary transformations.) Before completing the chapter with a study of some pertinent properties of the free groups, several examples of the use of the Tietze theorem are worked.

In Chapter 7 is found the only stylistic anomaly of the book. The material covered includes the free differential calculus. The motivation for the definitions is unexplicably omitted, and the Alexander Matrix itself is introduced without any reason given for its name. The usefulness of the Tietze theorem is however well illustrated here, as it is this theorem which implies that the presentation invariants introduced by means of the free calculus are actually invariants of the group. The free calculus provides, via the Alexander matrices, the means for constructing invariants of a group from a presentation. It is the application of this technique to the particular situations of knot theory which in fact is the main point of this book, and much material in the remaining chapters is pointed to this goal. Let us see how this is done.

In Chapter 1 one of course finds the basic definitions. These are inspired by an expressed need for a mathematical model of the physical situation of a knot, so that we find the tame (polygonal) theory placed before the wild, and the domain restricted to 3-space or the 3-sphere. The definition adopted for equivalence of knots is the following: Two knots are equivalent if a homeomorphism of the containing space throws one onto the other.

The possibility of using isotopy type as a definition of equivalence is also discussed.

Chapter 2 develops the fundamental group and takes up such questions as the effect of the base point, and the behavior of π_1 under continuous maps. The descriptions of the mathematical objects used in this chapter are very suggestive, and include such pictorial terms as the "path of a particle," "stopping time," "initial point," etc.

A complete and correct computation of the fundamental group of a circle ends this chapter.

The lines of thought begun in Chapters 1 and 2 are punctuated by Chapters 3 and 4. We shall return to the former in a moment.

After reading Chapter 5 the reader will find himself able to compute the fundamental group of a good many spaces. The chief tools made available are the van Kampen theorem (a proof of which is delayed till the Appendix), the ideas of deformation, retraction, deformation retract, and homotopy type. (Two spaces X and Y are defined to be of the same homotopy type if there exists a sequence of spaces X_i such that $X_0 = X$, $X_n = Y$ and X_i is a deformation retract of X_{i+1} or X_{i-1}.)

Chapter 6 deals with the special problem of computing the fundamental group of the complement of a knot. It is this group to which the ideas of Chapter 7 are addressed. In order to avoid a later discontinuity the proofs in Chapter 6 are somewhat more general than needed.

Returning to Chapter 3 we find the fundamentals for Chapter 4 developed. The material concerns the free groups, their definition and their universal property (the latter is a theorem in this book). It is interesting to note that the authors liken the free groups to a coordinate system in geometry, and a group defined by relations to a variety.

Summarizing then, we find Chapters 1, 2, 5, and 6 of a geometric nature, and Chapters 3, 4 and 7 algebraic. They are woven together as necessity dictates, so that for example the idea of a group presentation makes the computation of a fundamental group meaningful.

The culmination of the ideas of the book is reached in Chapter 8 where the polynomials of a knot are defined.

Beginning with a proof that an abelianized knot group is infinite cyclic, the authors thread their way through some neat algebra to show that they can legitimately define certain polynomials which are invariants of an equivalence class of knots. (The first of these polynomials is the classical Alexander Polynomial.) A number of examples (8) are worked and they illustrate nicely the strength and weakness of the invariants defined.

The final chapter, in which the Alexander Polynomial is characterized as being symmetric and equalling 1 when evaluated at 1, has for me a mysterious significance. The reason for this is the following:

The symmetry follows in this development from the existence of a pair of so called dual presentations of the knot group. (This proof is distinct from and independent of Milnor's duality theorem for Reidemeister torsion.) These dual presentations owe their existence to a certain geometric construction (in Chapter 6) and if one is of a geometric frame of mind one wonders how general this construction is. While from an algebraic point of view the question arises whether these dual presentations go far in characterizing a group as a knot group. In addition, the relation of this idea to any other in algebra is somewhat obscure to me.

Notwithstanding these personal feelings, the ninth chapter is quite clear and precise and makes its points without difficulty.

Finally the Appendix contains three items; a proof of van Kampen's theorem, a proof that differentiable knots are equivalent to polygonal ones, and some abstract nonsense about categories and groupoids which parallels the development of the fundamental group in Chapter 2.

The book ends with a "Guide to the Literature." This is intended to orient the student with respect to recent work in the field, and should prove extremely useful to those interested.

The Bibliography is quite complete.

The book is laced with exercises both difficult and easy.

There is practically no limitation on who can read this book. It is certainly suitable for use as a text in a graduate course at any level. It could be used to introduce talented high school seniors to topology. It can in fact be read with profit by practically any mathematician. The ideas he will not have seen before will be stimulating, while those with which he is familiar he will find well presented.

In short, this well-written, carefully designed and somewhat unusual book may be appreciated by mathematicians at many levels of development. Consequently, it is a very welcome addition to the mathematical literature.

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