A NEW CONSTRUCTION FOR HADAMARD MATRICES

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An Hadamard matrix $H$ is a square matrix of ones and minus ones whose row (and hence column) vectors are orthogonal. The order $n$ of an Hadamard matrix is necessarily 1, 2 or $4t$ with $t=1, 2, 3, \cdots$. It has been conjectured that this condition ($n=1, 2$ or $4t$) also insures the existence of an Hadamard matrix. Constructions have been given for particular values of $n$ and even for various infinite classes of values. While other constructions exist, those given by [1]-[7] exhaust the previously known values of $n$. This paper gives a new construction which yields, among others, the previously unknown value $n=156$, leaving only two undecided values of $n=4t \leq 200$ (these are 116 and 188).

An Hadamard matrix is said to be of the Williamson type if it has the structure imposed by Williamson [6], that is

$$H = \begin{bmatrix} A & B & C & D \\ -B & A & -D & C \\ -C & D & A & -B \\ -D & -C & B & A \end{bmatrix},$$

where each of $A, B, C, D$ is a symmetric circulant $t \times t$ matrix. Notice that if a Williamson type matrix exists for $n=4t$, then an Hadamard matrix (not obviously Williamson) of order $m=12t$ would exist provided one could find a $12 \times 12$ matrix with the following properties. Each row and column must contain precisely three $\pm A$'s, three $\pm B$'s, three $\pm C$'s, three $\pm D$'s and the rows must be formally orthogonal (i.e., $A, B, C, D$ are to be considered as independent quantities). We have discovered such a matrix and display it as Figure 1.

Among the known orders of Williamson type matrices [1], [6], only 52 yields a new value of $n$ by this construction. This gives an Hadamard matrix of order 156. For definiteness, the first rows of $A, B, C, D$ for one of the Williamson type Hadamard matrices of order 52 are given (here $+$ means $+1$ and $-$ stands for $-1$).

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\[ A \begin{align*} &+ + - - + - + - - - + \\ &+ - - - + + + + + - - \\ &+ + + - + - - + + + + \\ &+ + - + - + + + + + - + + \\ \end{align*} \]

\[ B \begin{align*} &A A A B -B C -C -D B C -D -D \\ &A -A B -A B -B -D D -C -B -D C -C \\ &A -B -A A -D D -B B -C -D C -C \\ &B A -A -A D D D C C -B -B -C \\ &B -D D D A A A C -C B -C B \\ \end{align*} \]


**Figure 1**

**References**


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