In this paper we show that if $K$ and $L$ are $n$-complexes, then $K$ and $L$ are isomorphic iff the 1-sections of the first derived complexes of $K$ and $L$ are isomorphic. This provides a low-dimensional method for establishing the isomorphism (homeomorphism) of complexes (polyhedra).

Throughout, $s_p$ will denote a (rectilinear) $p$-simplex with vertices $a^0, a^1, \ldots, a^p$; $K$ will denote a (finite geometric) complex with $n$-section $K^n$ and first derived complex $K'$. The closed star of a vertex $a$ of $K$, $\text{st}(a)$, is the set of simplexes of $K$ having $a$ as a face and all their faces. For more details see [2].

**Definition 1.** An $n$-complex $K$ is full provided, for any subcomplex $L$ of $K$ which is isomorphic to $s_p$, $2 \leq p \leq n$, $L^0$ spans a $p$-simplex of $K$.

**Theorem 1.** Suppose $K$ and $L$ are full $n$-complexes. Then $K$ and $L$ are isomorphic iff $K^1$ and $L^1$ are isomorphic.

**Proof.** We need only consider the case when $K^1$ and $L^1$ are isomorphic. Let $\nu: K^1 \rightarrow L^1$ be an admissible vertex transformation of $K^1$ onto $L^1$ with an admissible inverse. Then $a^0, a^1$ span a 1-simplex of $K$ iff $\nu(a^0), \nu(a^1)$ span a 1-simplex of $L$. Furthermore, for any $p$, $2 \leq p \leq n$, if $a^0, a^1, \ldots, a^p$ span a $p$-simplex $s_p$ of $K$, then $\nu(s_p)$ is isomorphic to $s_p$. So, using the fullness of $L$, we get that $(\nu(s_p))^0$
\[
\{v(a^0), v(a^1), \ldots, v(a^p)\} \text{ spans a } p\text{-simplex of } L. \text{ Similarly, if }
\{v(a^0), v(a^1), \ldots, v(a^p)\} \text{ spans a } p\text{-simplex of } L, \text{ then } \{a^0, a^1, \ldots, a^p\}
\text{ spans a } p\text{-simplex of } K. \text{ Hence, } v \text{ is an admissible vertex transformation of } K \text{ onto } L \text{ with an admissible inverse and so } K \text{ and } L \text{ are isomorphic.}
\]

**Lemma 1.** If \( K \) is an \( n \)-complex, then \( K' \) is a full \( n \)-complex.

**Proof.** Suppose \( L \) is a subcomplex of \( K' \) and \( L \) is isomorphic to \( s^p_n, 2 \leq p \leq n. \) Then there is a barycenter \( b \) of a \( q \)-simplex of \( K, p \leq q \leq n, \) such that \( L \subseteq st(b). \) Hence \( L^0 \) spans a \( p \)-simplex of \( K. \)

**Theorem 2.** If \( K \) and \( L \) are \( n \)-complexes, then \( K \) and \( L \) are isomorphic iff \((K')^1 \) and \((L')^1 \) are isomorphic.

**Proof.** Suppose \( K \) and \( L \) are isomorphic. Then \( K' \) and \( L' \) are isomorphic \( n \)-complexes. Since they are both full (Lemma 1) we can apply Theorem 1 to get that \((K')^1 \) and \((L')^1 \) are isomorphic.

Now assume that \((K')^1 \) and \((L')^1 \) are isomorphic. Then Theorem 1 implies \( K' \) and \( L' \) are isomorphic and so \( K \) and \( L \) are isomorphic (see [1]).

**References**


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