Consider the following system of equations:

\[
\sum_{j=1}^{M} \sum_{|\alpha| \leq m} a^{k,j}_{\alpha}(x, t) (\partial / \partial x)^{\alpha} u_j - \delta_{kj} (\partial / \partial t) u_k = f_k, \quad k = 1, \cdots, M
\]

here \( x \) is a point in \( E^n \) and \( t \in (0, R) \), \( R < \infty \).

Assume the system (1) is parabolic in the sense of I. G. Petrovsky, i.e. the roots \( \lambda(x, t; z) \) of the equation:

\[
\text{Det} \left( \sum_{|\alpha| = m} a^{k,j}_{\alpha}(x, t) (iz)^{\alpha} - \delta_{kj} \lambda \right) = 0
\]

satisfy \( \text{Re}(\lambda(x, t; z)) < -\delta < 0 \) for \( |z| = 1 \), independent of \( (x, t) \).

Define \( L_{\text{w},1}^M(E^n \times (0, \infty)) \) to be the closure in the class of distributions of \( E^{n+1} \) of the functions \( u \in C_0^\infty(E^n \times (0, \infty)) \) with respect to the norm:

\[
\|u\|_{m,1} = \sum_{|\alpha| \leq m} \left( \int_0^R \int_{E^n} \left| (\partial / \partial x)^{\alpha} u \right|^p \, dx \, dt \right)^{1/p}
\]

Define \( (L_{\text{w},1}^{p,m})^M \) to be all vectors \( u = (u_1, \cdots, u_M) \) with \( u_k \in L_{\text{w},1}^{p,m}(E^n \times (0, \infty)) \).

Concerning the coefficients of (1), assume that:

(i) \( a^{k,j}_{\alpha}(x, t) \) are bounded and measurable over \( E^n \times (0, R) \) for all \( \alpha, k, j \),

(ii) for \( |\alpha| = m \), \( a^{k,j}_{\alpha}(x, t) \) are uniformly continuous in \( E^n \times (0, R) \), for all \( k, j \).

**Theorem.** Given any vector-valued \( f = (f_1, \cdots, f_M) \), where \( f_k \in L^p(E^n \times (0, R)) \), there exists a unique \( u \in (L_{w,1}^{p,m})^M \) satisfying system (1).

The proof of this theorem is based upon the following representation of the operator \( L: (L_{w,1}^{p,m})^M \rightarrow (L^p)^M \) given by (1):

\[
Lu = (I + K) ((-1)^{m/2} \Delta^{m/2} + \partial / \partial t) I u,
\]
where $I$ is the identity matrix;

$$((-1)^{m/2} \Delta^{m/2} + \partial/\partial t)Iu = ((-1)^{m/2} \Delta^{m/2}u) + (\partial/\partial t)u_j,$$

and

$$\Delta u_j = \sum_{k=1}^{M} (\partial^2/\partial x_k^2) u_j,$$

$m$ is assumed to be an even number, and finally $K = K_1 + K_2$, $K_i = (K_i^{k,j}(x, t; y, s))$ where

$$K_i^{k,j}(u) = \lim_{\epsilon \to 0} \int_0^{t-s} \int_{B^n} K_i^{k,j}(x, t; x - y, t - s) u(y, s) \, dy \, ds,$$

for $u \in L^p(E^n \times (0, R))$.

Here $K_1$ is a matrix of singular integral operators, as defined in Abstract 65T-69, Notices Amer. Math. Soc. 12 (1965); while:

$$\|K_i\|_{1} = \text{ess sup}_{(x, t) \in E^n \times (0, R)} \sum_{k,j} \int_0^R \int_{B^n} |K_i^{k,j}(x, t; y, s)| \, dy \, ds < \infty.$$

Therefore $I + K$ is a bounded operator from $(L^p(E^n \times (0, R)))^M$ into itself.

The proof consists in showing that $I + K$ is actually an isomorphism from $(L^p(E^n \times (0, R)))^M$ onto $(L^p(E^n \times (0, R)))^M$ and hence the problem is reduced to studying the operator $((-1)^{m/2} \Delta^{m/2} + \partial/\partial t)I$.

The theorem establishes existence and uniqueness for generalized solutions with initial condition zero. For the general initial value problem, the decomposition (2) reduces the problem to the operator $((-1)^{m/2} \Delta^{m/2} + \partial/\partial t)I$, with the same initial condition.