This paper generalizes a result of Grothendieck [2] to the case of a nonabelian $W^*$-algebra.

**Theorem.** Let $M$ be a $W^*$-algebra and $N$ a $C^*$-subalgebra of $M$ which is separable in the norm topology. Then there exists a bounded projection of $M$ onto $N$ iff $N$ is finite-dimensional.

**Proof.** Suppose $P : M \to N$ is a bounded projection. If $N$ is infinite-dimensional, by [3] it must have an infinite-dimensional abelian $*$-subalgebra. Call it $N_0$. Then $N_0$ is isomorphic to $C_0(X)$, the continuous functions vanishing at infinity on some locally compact Hausdorff space $X$. Thus, by Urysohn's Lemma, there exists a sequence $\{b_k\}$ of orthogonal, positive elements of $N_0$ with $\|b_k\| = 1$ for all $k = 1, 2, \ldots$.

By the Hahn-Banach Theorem we may choose $\{f_k\}$ in $N^*$ with $\|f_k\| = 1$ and $f_k(b_2) = \delta_{kj}$. Since $N$ is separable, the unit ball $B$ of $N^*$ is weak* sequentially compact. Thus, taking subsequences if necessary, we may assume that $\{f_k\}$ is weak* convergent.

Now $P^* : N^* \to M^*$ is weak* continuous, so $\{P^*(f_k)\}$ is weak* convergent in $M^*$. Also $P^*(f_k)(b_j) = f_k(b_j) = \delta_{kj}$. But $\sum_{i=1}^\infty (b_k)$ exists in the $s$-topology of $M$, so Theorem III.7 of [1] applies to give $\sum_{i=1}^\infty f_j(b_k)$ exists uniformly for $j = 1, 2, \ldots$, a contradiction. Thus $N$ is finite dimensional.

The reverse implication is well known. Q.E.D.

**Bibliography**


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