A family of almost periodic functions on the reals, $\mathbb{R}$, to complex $n$-space, $C^n$, is compact in the uniform topology if and only if it is (a) closed, (b) uniformly bounded, (c) uniformly equicontinuous, and (d) uniformly almost periodic. This is a result of Bochner [1]. Of the above criteria, part (d) seems to be the most difficult to verify. We offer two results in this direction.

Recall that the family $A$ is uniformly almost periodic if for each $\epsilon > 0$, the set $T(A, \epsilon) = \{ \tau: |f(x+\tau)-f(\tau)| < \epsilon$ for all $x \in \mathbb{R} \}$ is relatively dense. For $A$ a singleton, this is Bohr's definition of an almost periodic function. Let $\exp(\phi)$ be the set of real numbers $\lambda$, such that $\lim_{T \to \infty} (1/T) \int_{0}^{T} \phi(x)e^{-\lambda x} dx = 0$. If $A$ is a compact family then $\exp(A) = \cap_{\phi \in A} \exp(\phi)$ is countable. Hence we will consider sets of the following form. Let $C(M, A) = \{ \phi | \phi \text{ is almost periodic}, \|\phi\| \leq M, \exp(\phi) \subset \Delta \}$ where $M$ is a fixed real number, $\|\cdot\|$ is the supremum norm, and $\Delta$ is a given countable set of reals.

**Theorem 1.** If $\Delta$ has no finite limit point, then any uniformly equicontinuous family in $C(M, \Delta)$ has compact closure.

**Theorem 2.** If $A \subset C(M, \Delta)$ is the family that is uniformly Lipschitz, i.e.: there is a $K > 0$, such that $f \in A$ if and only if $|f(t) - f(s)| \leq K|t - s|$ for all $t$ and $s$, then $A$ has compact closure if and only if $\Delta$ has no finite limit point.

In fact, if $\Delta$ has no finite limit point, then $A$ is a convex compact set having the fixed point property.

If $\Delta$ has no finite limit point, then let $\Delta = \{ \lambda_n \}$ with $|\lambda_1| \leq |\lambda_2| \leq \cdots$. By a result of Bredhina [2], there exist polynomials $\sigma_n(f, x) = \sum_{k=-1}^{n} a_k(f)e^{\lambda_k x}$ such that $\|f - \sigma_n(f)\| \leq 10 \omega_f(1/|\lambda_n|)$ where $\omega_f(x) = \sup\{ |f(y) - f(z)| : |y - z| \leq x \}$. For any $\epsilon > 0$, the polynomials $\{\sigma_n(f)\}_{f \in A}$ are an $\epsilon$-net for some $n_\epsilon$. This collection is uniformly almost periodic; hence so is $A$.

If the family $A$ is Lipschitz and $\Delta$ has a finite limit point, one constructs a closed ball of an infinite dimensional space in $A$. These are not compact. One uses the result of Bochner [3] to the effect that if $\exp(\phi)$ is a bounded set, then $\phi'$ exists and $\|\phi'\| \leq T\|\phi\|$ where $T$ is a
bound for exp(\phi). That is, for such functions, the Lipschitz condition is automatically satisfied.

Details of the above outline appear in [4] where an application of the fixed point property is also given.

REFERENCES


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