There are two very natural notions of equivalence of flows (see [1], [2]) on a manifold. One is the existence of a homeomorphism mapping orbits onto orbits, preserving the natural orientation of orbits but not necessarily their natural parametrisation. The second requires that the homeomorphism alter natural parametrisations by at most a positive constant multiple. We call the first relation on flows orbit-equivalence and the second flow-equivalence. There are obvious localisations of these relations. In general flow-equivalence is strictly stronger than orbit-equivalence. However, it is a consequence of the theorem of Hartman [5], [6], [7] and Grobman [3], [4] that the local notions of equivalence are the same at elementary (see [1], [2]) rest-points. The purpose of this note is to announce a similar result for elementary cycles. M. Shub has informed me that he and C. Pugh have also obtained this result.

1. Preliminaries. Let \( \phi: \mathbb{R} \times X \rightarrow X \) be a \( C^1 \) flow on a \( C^\infty \) manifold \( X \). We write \( \phi_\ast(t) = \phi(t, x) = \phi(t_\ast x) \), so that, for fixed \( x \in X \), \( \phi_\ast: \mathbb{R} \rightarrow X \) is \( C^1 \) and, for fixed \( t \in \mathbb{R} \), \( \phi_\ast: X \rightarrow X \) is a \( C^1 \) diffeomorphism. Let \( U \) be open in \( X \). For fixed \( x \in U \) let \( I_x \) denote the component of \( \{ \phi_\ast \} \) containing \( 0 \), and let \( D_U \) denote \( \bigcup_{x \in U} I_x \times \{ x \} \).

Now suppose that \( \Psi \) is a \( C^1 \) flow on a \( C^\infty \) manifold \( Y \) and that \( A \) and \( B \) are subsets of \( X \) and \( Y \) respectively. We say that \( A \) is flow-equivalent to \( B \) (with respect to the given flows) if there exist open neighbourhoods \( U \) of \( A \) and \( V \) of \( B \) and a homeomorphism \( h: U \rightarrow V \) such that \( h(A) = B \) and, for all \( (t, x) \in D_U \),

\[
h\phi(t, x) = \Psi(\alpha(t), h(x)),
\]

where \( \alpha: \mathbb{R} \rightarrow \mathbb{R} \) is a multiplication by some positive constant. In this case \( h \) maps orbit components of \( \phi \) in \( U \) onto orbit components of \( \Psi \) in \( V \), preserving orientation.

Let \( \nu \in GL(E) \), where \( E \) is a finite dimensional real normed linear space. Let \( F \) be the largest invariant subspace of \( E \) on which \( \nu \) has no complex eigenvalues of modulus 1. We call \( \nu|F \) the hyperbolic part of \( \nu \).

Recall [8] that we may associate with any hyperbolic linear auto-
morphism of a $k$-dimensional real normed linear space its suspension, which is a flow on a $(k+1)$-dimensional manifold. It has precisely one cycle, corresponding to the unique periodic point 0 of the automorphism.

2. Results. The following is an analogue, for cycles, to the theorem of Hartman and Grobman for rest-points:

**Theorem 1.** Let $\phi$ be a $C^1$ flow on $X$, and let $C$ be an elementary cycle of $\phi$. For any $x \in C$, let $D$ be the unique cycle of the suspension of the hyperbolic part of $T_x \phi^r$, where $r$ is the period of $C$. Then $C$ is flow-equivalent to $D$.

As corollaries one deduces

**Theorem 2.** Let $C$ and $D$ be elementary cycles of $C^1$ flows. Then $C$ is flow-equivalent to $D$ if and only if $m(C)=m(D)$, $n(C)=n(D)$, and $m^-(C)-m^-(D)$ and $n^-(C)-n^-(D)$ are even, where $m$, $n$, $m^-$ and $n^-$ denote the numbers of expanding, contracting, real negative expanding and real negative contracting characteristic multipliers.

and

**Theorem 3.** Elementary cycles of $C^1$ flows are flow-equivalent if and only if they are orbit-equivalent.

The proof of Theorem 1 reduces, in effect, to a proof that there exists, at $x$, a local cross-section that is invariant under $\phi^r$. By taking a suitable chart and using a bump-function we may reduce this to the following

**Lemma 4.** Let $\nu: E \to E$ be a hyperbolic linear automorphism, let $f: E \to E$ be a homeomorphism and let $\xi: E \to \mathbb{R}$ be a linear map. Suppose that, for some $d \geq 0$, $f - \nu$ and $\xi$ vanish at 0 and on $\{x \in E; \|x\| \geq d\}$ and are Lipschitz, the former with constant $\kappa$. Then, if $\kappa$ is sufficiently small, there exists a continuous map $\theta: E \to \mathbb{R}$ such that $\theta = \theta f + \xi$.

Subject to the condition $\theta(0) = 0$, the map $\theta$ is uniquely defined on the stable and unstable manifolds of the origin with respect to $f$. Elsewhere, however, it is not unique. In the proof [9] of the lemma an explicit map $\theta$ is constructed.

**References**


4. ———, *Topological classification of the neighborhood of a singular point in n-dimensional space*, Mat. Sb. 56 (98) (1962), 77–94. (Russian)


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