A NORMAL SPACE X FOR WHICH X × I IS NOT NORMAL

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In [1] C. Dowker gave a number of interesting characterizations of normal Hausdorff spaces whose cartesian product with the closed unit interval is not normal. Thus, such a space is often called a Dowker space; a Dowker space X will be described below. It was previously known only that the existence of a Dowker space is consistent with the usual axioms of set theory [2], [3]. The proof that X is a Dowker space uses no set theoretic assumptions beyond the axiom of choice.

We use the convention that an ordinal \( \lambda \) is the set of all ordinals less than \( \lambda \). An ordinal \( \gamma \) is said to be cofinal with \( \lambda \) if there is a subset \( \Gamma \) of \( \lambda \) order isomorphic with \( \gamma \) such that \( \alpha \in \lambda \) implies \( \alpha \leq \beta \) for some \( \beta \in \Gamma \); let \( \text{cf}(\lambda) \) be the smallest ordinal cofinal with \( \lambda \).

Define \( F = \{ f : \omega_0 \to \omega_0 \mid \forall n \in \omega_0, f(n) \leq \omega_n + 1 \} \).

Define \( X = \{ f \in F \mid \exists k \in \omega_0 \text{ such that } \forall n \in \omega_0, \omega_0 < \text{cf}(f(n)) < \omega_k \} \).

For \( f \) and \( g \) in \( F \), define \( U_{fg} = \{ h \in X \mid \forall n \in \omega_0, f(n) < h(n) \leq g(n) \} \). Then topologize \( X \) by using the set of all \( U_{fg} \) for \( f \) and \( g \) in \( F \) as a basis for the topology. The resulting space is a collectionwise normal Dowker space.

REFERENCES


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