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THE CONVERSE TO THE FIXED POINT THEOREM OF P. A. SMITH[1]

BY LOWELL JONES

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$\boldsymbol{Z}_n$, Z_n will denote the multiplicative, and additive cyclic groups of order n.

Recall that a simplicial complex K is a Z_n-homology manifold if the link of every simplex in K is a Z_n-homology sphere. A Z_n-homology manifold pair $(K, \partial K)$ is defined similarly. A group action $G \times X \to X$ on the space X is *semifree* if, for each $x \in X$, G acts either trivially or freely on the orbit of x.

Let n be an even integer, and $(K, \partial K) \subset (D^m, \partial D^m)$ a combinatorial embedding having even codimension $\geqq 6$ and further satisfying $K \cap \partial D^m = \partial K$.

THEOREM. *$K \subset D^m$ is the fixed point set of a semifree, combinatorial group action $\boldsymbol{Z}_n \times D^m \to D^m$ if and only if*

(1) $\bar{H}(K, Z_n) = 0$,

(2) *$(K, \partial K)$ is a Z_n-homology manifold pair.*

The "only if" part of the theorem was proven by P. A. Smith [**3**].

The theorem holds for n odd provided the regular neighborhood for $(K, \partial K)$ in $(D^m, \partial D^m)$ admits a one-parameter cross section, e.g., $(K, \partial K) \subset (D^m, \partial D^m)$ factors as $(K, \partial K) \subset (D^{m-2}, \partial D^{m-2}) \subset (D^m, \partial D^m)$.

A classification theorem can also be proven, which gives a bijective correspondence between equivalence classes of semifree $\boldsymbol{Z}_n$-actions on D^m having $K \subset D^m$ for fixed point set and the elements of $\boldsymbol{H}_0(K, \partial K)$, where $\boldsymbol{H}_*(\ )$ is a certain computable homology functor.

These results extend to the following situations:

(1) when M replaces D^m, where M is a simply connected manifold satisfying $\bar{H}(M, Z_n) = 0$,

(2) a relative version of (1),

(3) replacing $\boldsymbol{Z}_n$ by any finite group which acts freely on the sphere normal to K in D^m.

In order to prove the above results the author has been led to

(a) the extension of the Characteristic Variety Theorem [**4**] to the nonsimply connected case (see [**1**]), and

(b) the extension of transversality and surgery techniques to the Poincaré duality category (see [**2**]).

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DEPARTMENT OF MATHEMATICS, UNIVERSITY OF CALIFORNIA, BERKELEY, CALIFORNIA 94720