

THE CONVERSE TO THE FIXED POINT THEOREM OF P. A. SMITH¹

BY LOWELL JONES

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Z_n , Z_n will denote the multiplicative, and additive cyclic groups of order n .

Recall that a simplicial complex K is a Z_n -homology manifold if the link of every simplex in K is a Z_n -homology sphere. A Z_n -homology manifold pair $(K, \partial K)$ is defined similarly. A group action $G \times X \rightarrow X$ on the space X is *semifree* if, for each $x \in X$, G acts either trivially or freely on the orbit of x .

Let n be an even integer, and $(K, \partial K) \subset (D^m, \partial D^m)$ a combinatorial embedding having even codimension ≥ 6 and further satisfying $K \cap \partial D^m = \partial K$.

THEOREM. $K \subset D^m$ is the fixed point set of a semifree, combinatorial group action $Z_n \times D^m \rightarrow D^m$ if and only if

- (1) $\bar{H}(K, Z_n) = 0$,
- (2) $(K, \partial K)$ is a Z_n -homology manifold pair.

The “only if” part of the theorem was proven by P. A. Smith [3].

The theorem holds for n odd provided the regular neighborhood for $(K, \partial K)$ in $(D^m, \partial D^m)$ admits a one-parameter cross section, e.g., $(K, \partial K) \subset (D^m, \partial D^m)$ factors as $(K, \partial K) \subset (D^{m-2}, \partial D^{m-2}) \subset (D^m, \partial D^m)$.

A classification theorem can also be proven, which gives a bijective correspondence between equivalence classes of semifree Z_n -actions on D^m having $K \subset D^m$ for fixed point set and the elements of $H_0(K, \partial K)$, where $H_*()$ is a certain computable homology functor.

These results extend to the following situations:

- (1) when M replaces D^m , where M is a simply connected manifold satisfying $\bar{H}(M, Z_n) = 0$,
- (2) a relative version of (1),
- (3) replacing Z_n by any finite group which acts freely on the sphere normal to K in D^m .

In order to prove the above results the author has been led to

- (a) the extension of the Characteristic Variety Theorem [4] to the nonsimply connected case (see [1]), and
- (b) the extension of transversality and surgery techniques to the Poincaré duality category (see [2]).

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DEPARTMENT OF MATHEMATICS, UNIVERSITY OF CALIFORNIA, BERKELEY, CALIFORNIA 94720