(if $u$ is a potential then $u = \int \mathcal{G}\, e^u(da)$ where $\mathcal{G}$ is the Newtonian Green function) and $m(da)$ is a fixed speed measure, in complete analogy with the case $d = 1$. This chapter is a "tour de force" with few references to the earlier ones.

The final chapter, 8, returns to the study of a general diffusion, but unlike the other chapters it is rather a statement of problems and principles than an empirically organized collection of facts. The language is that of road maps and speeds, due to W. Feller, but several of the major problems remain unsolved at the present time (it even may be doubted if all of them admit of complete solutions). On the other hand, the continuity of path assumption is used effectively to condense some of the results of probabilistic potential theory, and especially for the proof that the natural capacities of G. A. Hunt are the same for the forward and reversed (adjoint) processes. The chapter terminates with a short introduction to the general boundary theory of diffusions, which has undergone much development since publication,

Sufficient time may by now have elapsed to make it appropriate to give an overview of this extraordinary work. Suppose that one would take a trip on an ideal diffusing particle—where would one go and what would one see? The trip is rough and often confusing. In many cases one cannot see the route because of the superabundance of detail in the landscape. Furthermore, the trip has no clear beginning or final conclusion. Nonetheless, it does describe a significant branch of mathematics with an elegance of taste and a finality unlike any other work on the subject. Whether or not one wishes to make the journey, it is available and the subject is strengthened by its presence. One might, however, express a hope that in a new edition the authors would provide a few more landmarks.

FRANK B. KNIGHT

*Celestial Mechanics.* I, II by Shlomo Sternberg. Vol. I, 158 pp., Vol. II, 304 pp. W. A. Benjamin, Inc., New York, 1969. $7.95.

In recent years there has been considerable growth of interest among mathematicians in classical mechanics. The article of S. Smale, *Topology and mechanics*, and S. Mac Lane's survey in the Monthly are indicative of this.

Sternberg's book has one important virtue: The author deals not only with generalities (such as symplectic structures and so on) but also treats details of difficult concrete problems.

The theory of Hamiltonian perturbations of quasiperiodic motions in celestial mechanics provides good examples of such concrete problems. For a long time, following the classical work of H. Poincaré and G. D. Birkhoff, the difficulties of divergence of the perturbation series, connected with resonances (so called "small divisors") seemed an unavoidable obstruction to all progress.

The first to make a dent in these problems was C. L. Siegel (1942), but the problems became tractable only after Kolmogorov (1954) introduced an iteration method, reminiscent of "Newton's method" in elementary calculus, to deal with small divisors.

The book of Sternberg is based on notes from a course given at Harvard in 1968. Its main purpose is to examine recent developments in celestial mechanics beginning with this work of Kolmogorov.

The first part of the book is devoted to the history of the subject: It discusses the influence of celestial mechanics on the development of mathematics. Among other things, it contains H. Weyl's solution of Lagrange's problem of the mean motion of the periheliums of Earth and Venus, the proofs of the ergodic theorems, the Peter-Weyl theorem and Bohr's theorem on almost periodic functions. In addition, it contains a short discussion of the history of Kepler's discoveries, of Delaunary variables and of Hill's life and his lunar theory, and ends with a discussion of some of the astronomical implications of general relativity.

The main part of the book is Chapter 3, which is devoted to implicit function theorems of the type used by Nash for the imbedding of Riemannian manifolds in Euclidean space and by Kolmogorov in celestial mechanics. The author follows the Varenna lectures of Moser with some new details added and numerous applications (including the canonical form theorem for differential equations on the torus, Siegel's linearization theorem, and Kolmogorov's theorem on analytical area preserving maps of the annulus).

The book ends with a chapter on the classical reductions of the 3-body problem (elimination of nodes and so on) in the spirit of Poincaré's *Méthodes nouvelles.* This allows the author to make some explicit applications of the general theory developed in Chapter 3.

In spite of the important advances made in Hamiltonian mechanics in recent years, the number of mathematicians working in this area is still small. One can hope that Sternberg's book will attract more mathematicians to these important problems which are very difficult, perhaps too difficult, for modern mathematics.

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