UNIQUENESS OF ORIENTATION PRESERVING PL INVOLUTIONS OF 3-SPACE

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1. Introduction. Waldhausen [2] has proven that every PL involution of $S^3$ with 1-dimensional fixed point set is PL equivalent to the one which rotates $S^3$ around an unknotted simple closed curve. In this note we show how the corresponding result for $R^3$ (which has been heretofore unknown) follows from a technique used by the second author in his recent paper [1]. Specifically, we prove

THEOREM 1. Every orientation preserving PL involution of $R^3$ is PL equivalent to the one which rotates $R^3$ around the z-axis.

Since such an involution must have 1-dimensional fixed point set, the above theorem is a consequence of the following theorem if one considers the one-point compactification of $R^3$.

THEOREM 2. Let $h$ be an involution of a closed 3-manifold $M$ with 1-dimensional fixed point set $F$. If, for some $x \in F$, there exists a triangulation of $M - \{x\}$ making $h|M - \{x\}$ piecewise linear, then there exists a triangulation of $M$ making $h$ piecewise linear.

Theorem 2 will be proved by literally imitating the reduction method [2, proof of Lemma 2] of Tollefson.

2. Proof of Theorem 2.

LEMMA. Let $M, F, x$ and $h$ be as in Theorem 2. Then, for any neighborhood $U$ of $x$, there exists in $U$ an invariant 3-cell $D$ containing $x$ in its interior such that $\partial D \cap F \neq \emptyset$ and $\partial D$ is a PL subspace of $M - \{x\}$.

PROOF. We indicate how to modify the proof of Lemma 2 of [1] to produce the desired invariant 3-cell $D$. We may assume that $F$ is not contained in $U$. Let $\Sigma$ be the set of all PL 2-spheres in $M - \{x\}$ that bound 3-cell neighborhoods of $x$ in $U$ and are in $h$-general position modulo $F$ (in the sense of [1]). The lemma follows from the proof of Lemma 2 of [1] if the phrase "2-spheres not bounding 3-cells" is replaced by "PL 2-spheres in $M - \{x\}$ bounding 3-cell neighborhoods of $x$ in $U".

In order to prove Theorem 2, consider a sequence of invariant 3-cells
(as in the lemma) $D_1, D_2, \ldots$ such that $D_{i+1} \subset \text{Int}(D_i)$ and $\bigcap D_i = x$. Observe that $F$ meets each 2-sphere $\partial D_i$ in two points. By applying the above-mentioned result of Waldhausen to $h\|D_i - \text{Int}(D_{i+1})$ (for each $i$), we find that $h\|D_i$ is topologically equivalent to the cone of $h\|\partial D_1$. Now extend the triangulation of $M - \text{Int}(D_1)$ to $M$ by triangulating $D_1$ as the cone over $\partial D_1$. This proves Theorem 2.

REFERENCES


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