We announce here some results of a paper to appear elsewhere [1].

Let a torus $T$ act continuously on a topological space $X$. Let $X 	o X_T \to B_T$ be the fibre bundle with fibre $X$ associated (by means of the action of $T$ on $X$) to the universal principal $T$ bundle $T \to E_T \to B_T$. We define the equivariant cohomology ring $H^*_T(X) = H^*(X_T)$ where $H^*$ denotes Čech cohomology with rational coefficients. When $Y$ is an invariant subspace of $X$, we define $H^*_T(X, Y) = H^*(X_T, Y_T)$. Then $R = H_T^*(B_T)$ is a polynomial ring and $H^*_T(X, Y)$ is a module over $R$ by means of $\pi^*$.

For each subtorus $L$ of $T$ let $P_L$ be the kernel of $H^*(B_T) \to H^*(B_L)$. Let $X^L = F(L, X)$ be the set of points fixed by $L$. We will assume that $X$ is compact. Given a closed invariant subspace $Y \subseteq X$ and an element $x \in H^*_T(Y)$, we define

$$I_x = \{a \in R \mid ax \text{ lies in the image of } H^*_T(X) \to H^*_T(Y)\}, \quad \text{and}$$

$$I^L_x = \{a \in R \mid ax \text{ lies in the image of } H^*_T(X^L \cup Y) \to H^*_T(Y)\}.$$

When $L \subseteq K$ are subtori, $I_x \subseteq I^L_x \subseteq I^K_x$. We say that $K$ belongs to $x$ if $K$ is maximal with respect to the property $I^K_x \neq R$.

1. **Theorem.** The isolated primary components of the ideal $I_x$ are the ideals $I^K_x$ where $K$ belongs to $x$. The radical of $I^K_x$ is $PK$, hence $\sqrt{I_x} = \bigcap PK$ where $K$ ranges over the subtori belonging to $x$.

2. **Corollary.** If $I_x$ is principal, the subtori belonging to $x$ are all of corank 1 and $I_x = \bigcap I^K_x$ where $K$ ranges over the subtori belonging to $x$. For each such $K$, $I^K_x = (\omega^d)$ where $d \geq 1$ and $\omega \in H^2(B_T)$ generates $PK$.

Assume that the fixed point set $F$ of the $T$ action on $X$ is not connected. Let $F = F^1 + \cdots + F^s$ be the connected components of the fixed point set, $s \geq 2$. We say that a subtorus $L$ connects $F^1$ and $F^2$ if they lie in the same component of $X^L$. We assume that $\dim H_T^*(X)$ is finite.

3. **Theorem.** Let $N \subseteq H_T^*(X)$ be the ideal generated by odd degree and $R$ torsion elements. Assume that $H^*_T(X)/N$ is generated by $k$ elements as an $R$ algebra. Then for every maximal subtorus $K$ connecting $F^1$ and $F^2$, $\rank K \geq \rank T - k$.  

4. Remark. This generalizes a result of Hsiang [3] that $F$ is connected whenever $H^*_R(X)$ is generated as an $R$ algebra by odd degree and $R$ torsion elements.

The following proposition is a technical result related to a theorem of Golber [2].

5. Proposition. Assume that $\dim H^*(X) = \dim H^*(F) < \infty$. Let $S = \{ x \in X \mid \text{rank } T_x \geq \text{rank } T - 1 \}$. Then the homomorphism $H_*(X, F) \to H_*(S, F)$ is injective.

We use the notation $X \sim Y$ to indicate that there is an isomorphism of rational cohomology rings $H^*(X) = H^*(Y)$. When $X \sim S^{k_1} \times \cdots \times S^{k_n}$ where the $k_i$ are odd integers, we define $e(X)$ to be the second symmetric polynomial $\sum_{i<j} (k_i + 1)(k_j + 1)$. If $X \sim S^{d_1} \times \cdots \times S^{d_s}$ where the $d_i$ are odd integers, for every subtorus $L$ of $T$ [3]. Hence $e(X^L)$ is defined. Further we define $g(X) = e(X) - e(F) - \sum_L [e(X^L) - e(F)]$ where $L$ ranges over the corank 1 subtori. For each subtorus $H$ of corank 2, we define $g(X^H)$ by using the induced $T/H$ action on $X^H$.

6. Proposition. $g(X) = \sum_H g(X^H)$ where $H$ ranges over the corank 2 subtori.

7. Remark. Golber [2] has proved that $g(X) = \sum g(X^H)$ when $X \sim S^{k_1} \times S^{k_2}$ where the $k_i$ are odd, and $F = \emptyset$.

When $X$ is a compact rational cohomology manifold and $F = F^1 + \cdots + F^s$ are the components of the fixed point set, let $f_i$ be a generator of the top dimensional cohomology group of $F^i$. After including $f_i \in H^*(F^i) \subset H^*(F) \subset H^*(F)$, we can define the ideal $I_{f_i}$. The following result was conjectured by Hsiang. It is a kind of splitting principle or Schur lemma for torus actions.

8. Theorem. The ideal $I_{f_i}$ is principal with a generator of degree $\dim X - \dim F^i$. This generator splits as a product of linear factors in $R$ corresponding to the subtori belonging to $f_i$.

Here $n = \dim X$ means that $H^n(X)$ is the top dimensional nonzero cohomology group of $X$. We do an explicit computation of $I_{f_i}$, when $X \sim$ quaternionic projective $n$ space [1].

9. Remark. Theorem 8 holds for torus actions on Poincaré duality spaces. It also holds for actions of $\mathbb{T}$-tori on Poincaré duality spaces over $\mathbb{Z}_p$. The Borel formula (see [3]) also holds for such actions [5].

Theorem 8 yields the following result of Hsiang and Su [4].

10. Theorem. When $X$ is a compact rational cohomology manifold and $X \sim QP^n$, quaternionic projective $n$ space, and a torus of rank $\geq 2$ acts
effectively on $X$, the fixed point set has at most one component $\sim QP^k$ with $k \geq 1$.

The results announced here also hold for actions of $p$-tori using $\mathbb{Z}_p$ cohomology.

REFERENCES


DEPARTMENT OF MATHEMATICS, UNIVERSITY OF KANSAS, LAWRENCE, KANSAS 66044

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF CALIFORNIA, BERKELEY, CALIFORNIA 94720

Current address (Tor Skjelbred): The Institute for Advanced Study, Princeton, New Jersey 08540