SOME EXTENSION THEOREMS FOR REGULAR MAPS OF STEIN MANIFOLDS

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The central result of this paper is an analogue, in the category of complex manifolds and holomorphic maps, of the tubular neighborhood theorem. The following theorems are proved.

**Theorem A.** Let $S$ be a Stein manifold and let $f: S \to M$ be a holomorphic embedding. Let $K \subset S$ be compact and let $N_f$ be the normal bundle of $f$. We identify $S$ with the zero section of $N_f$. Then there is a neighborhood $U$ of $K$ in $N_f$ and a holomorphic embedding $F: U \to M$ such that $F|U \cap S = f$.

**Theorem B.** The word "embedding" can be replaced by the word "immersion" in the above theorem.

**Theorem C.** Let $f: S \to M$ be a holomorphic map, where $S$ is Stein, such that $f$ is regular at $x_0 \in S$. Let $K \subset S$ be compact with $x_0 \in K$. Then there is a trivial bundle $\tilde{A}$ over $S$, $\dim C\tilde{A} = \dim CM$, a neighborhood $U$ of $K$ in $\tilde{A}$, and a holomorphic map $F: U \to M$ such that $F|U \cap S = f$ and $F$ is regular at $x_0$ (again $S$ is identified with the zero section of $\tilde{A}$).

When $M$ above is itself a Stein manifold, Theorems A and B are known and were proved by Forster and Ramspott. So the main effort of our work is to construct a strictly plurisubharmonic function $\phi$ on a neighborhood of $f(K)$ (in Theorem A) such that $\phi^{-1}((\infty, b])$ is compact for all $b \in R$. Theorem B follows then from Theorem A by a well-known result. Theorem C follows from Theorem A by embedding a neighborhood of $g(K)$ in $S \times M$ in some $C^q$ ($g(x) = (x, f(x))$) and using the holomorphic retraction theorem and other standard results.

A special case of Theorem A, namely the case where $S$ is a disc, can


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1223
be applied to show that the differential metric associated to the Kobayashi metric on a complex manifold is upper semicontinuous.

BIBLIOGRAPHY


