1. Shape. We give a solution to the following

Problem. Give necessary and sufficient conditions for a compactum $Z$ to have the shape of (A) a complex or (B) a finite complex.

Problem B makes sense in Borsuk's shape theory for compacta [2] but in order to give meaning to Problem A, we must extend Borsuk's theory to include noncompact complexes. A particularly simple treatment is in [7]. Alternatively one can replace "complex" by "ANR" in Problem A, and use Fox’s extension to metric spaces [9].

It is desirable that the conditions in Problems A and B be intrinsic. The following partial solution to Problem B is in [10]: a finite-dimensional 1-UV compactum has the shape of a finite complex if and only if its Čech cohomology with integer coefficients is finitely generated. But without the hypothesis 1-UV, the condition offered in [10] is not an intrinsic one.

Now for our solution. First some notation. If $(Z, z)$ is a pointed connected compact subset of a euclidean space $E$, let $\{(X_\alpha, z)\}$ be the inverse system of all connected open neighborhoods of $Z$ in $E$, pointed by $z$ and bonded by inclusion. Regarding $\{(X_\alpha, z)\}$ as an object of pro-# let $\text{pro} \pi_k(Z, z)$ be the pro-group $\{\pi_k(X_\alpha, z)\}$; let $\tilde{\pi}_k(Z, z)$ be its inverse limit (the $k$th shape group of $(Z, z)$). Let $\tilde{K}^0(G)$ denote the reduced projective class group of the group $G$ (see p. 64 of [12]).

**Theorem 1** [8]. Let $(Z, z)$ be as above. The following are equivalent:

(i) $\text{pro} \pi_k(Z, z)$ is isomorphic to $\tilde{\pi}_k(Z, z)$ in pro-groups for each $k \geq 1$; (ii) $(Z, z)$ has the pointed shape of a pointed complex of dimension max $\{3, \dim Z\}$; (iii) $(Z, z)$ is dominated in pointed shape by a pointed finite complex; (iv) $(Z, z)$ is movable and the natural topology on $\tilde{\pi}_k(Z, z)$ is discrete for each $k \geq 1$; (v) $(Z, z)$ is a pointed FANR. Furthermore, $Z$ has the shape


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of a finite complex if and only if an intrinsically defined "Wall obstruction" 
\[ w(Z, z) \in \tilde{K}^0(\tilde{\pi}_1(Z, z)) \] vanishes. All possible Wall obstructions occur among two-dimensional compacta.

Movable is defined in [4]; (iv) is explained in [6]; FANR is defined in [3]. Note that if \( \tilde{\pi}_1(Z, z) \) is free or free abelian, \( w(Z, z) = 0 \).

2. Pro-homotopy. Shape is the "inverse limit" of pro-homotopy and the above results are proved by means of new theorems in pro-homotopy: among them a Whitehead theorem (Theorem 2) and a stability theorem (Theorem 3).

If \( C \) is category, let pro-\( C \) be the category whose objects are inverse systems in \( C \) indexed by directed sets, and whose morphisms are as described in the Appendix to [1]. An object of pro-\( C \) indexed by the natural numbers is a tower in \( C \). Let \( CW_0 \) be the category of pointed connected (CW) complexes and pointed maps; let \( H_0 \) be the corresponding homotopy category. We suppress base points. If \( X = \{X_\alpha\} \) is in pro-CW\( _0 \) or pro-H\( _0 \), CW-dim \( X = \sup\{\dim X_\alpha\} \); h-dim \( X = \inf\{\text{CW-dim } Y | Y \text{ is isomorphic to } X \text{ in pro-} H_0 \} \). \( X \) is compact if each \( X_\alpha \) is a finite complex. \( \pi_k(X) \) is the pro-group \( \{\pi_k(X_\alpha)\} \); \( \tilde{\pi}_k(X) \) is its inverse limit group. A weak equivalence is a morphism inducing isomorphisms on \( \pi_k \) for all \( k \geq 1 \).

Theorem 2 is an extension of results in [11].

**Theorem 2** [8]. Let \( g: X \to Y \) be a morphism of pro-CW\( _0 \) and let \( n = \max\{1 + \text{CW-dim } X, \text{CW-dim } Y\} < \infty \). Suppose \( g#: \pi_k(X) \to \pi_k(Y) \) is an isomorphism for \( k \leq n \) and has a right inverse for \( k = n + 1 \). Then \( g \) induces an isomorphism of pro-H\( _0 \). If \( X \) and \( Y \) are towers, \( g \) need only be a morphism of pro-H\( _0 \).

Theorem 3 uses Theorem 2 together with [12].

**Theorem 3** [8]. Let \( X \) be a tower in \( H_0 \). (i) There exist a pointed complex \( Q \) and a weak equivalence \( q: Q \to X \) in pro-CW\( _0 \) if and only if \( \pi_k(X) \) is isomorphic in pro-groups to \( \tilde{\pi}_k(X) \) for all \( k \geq 1 \). In case (i) holds we have: (ii) \( Q \) can have dimension \( \max\{3, \text{h-dim } X\} \) and if \( \text{h-dim } X = 1 \), \( Q \) can be a bouquet of circles; (iii) if CW-dim \( X < \infty \), \( q \) induces an isomorphism in pro-\( H_0 \); (iv) if CW-dim \( X < \infty \) and \( X \) is compact, \( Q \) is dominated in \( H_0 \) by a finite complex, and \( X \) is isomorphic to a (pointed) finite complex \( P \) if and only if an intrinsically defined "Wall obstruction" \( w(X) \in \tilde{K}^0(\tilde{\pi}_1(X)) \).
vanishes: if \( w(X) = 0 \), \( \dim P = \dim Q \); (v) all possible Wall obstructions occur among towers of \( CW \)-dim 2.

REFERENCES


