Throughout this note let $G$ be an arbitrary discrete amenable group. Let $(\Omega, M, \lambda)$ be a probability space. Let $\mathcal{A}$ be the automorphism group of $(\Omega, M, \lambda)$. Let $T: G \rightarrow \mathcal{A}$ be a group homomorphism. We call $T$ an action of $G$ on $\Omega$. For each $g \in G$, let $T^g$ be the image of $g$ in $\mathcal{A}$ under $T$. Then $T^g$ is a measurable, measure-preserving, invertible map from $\Omega$ to itself.

If $Q$ is a partition of $\Omega$ and $\omega \in \Omega$, let $Q(\omega)$ be the element of $Q$ which contains $\omega$. If $E$ is a set let $|E|$ denote the cardinality of $E$.

Let $K$ be a subgroup of $G$. A net $\{A_\alpha\}$ of finite nonempty subsets of $K$ is said to satisfy property $P$ with respect to $K$ if $\lim_{\alpha} |A_\alpha|^{-1}|gA_\alpha \cap A_\alpha| = 1, g \in K$. (Since $K$ is amenable, such a net $\{A_\alpha\}$ exists; see [3].)

Let $P$ be a measurable partition of $\Omega$ with finite entropy. If $E$ is a finite nonempty subset of $G$, let $h_P(E) \in L^1(\Omega)$ be defined as follows:

$$h_P(E)(\omega) = -\log \lambda \left[ \left\{ \bigvee_{g \in E} (T^g)^{-1}P \right\} (\omega) \right], \quad \omega \in \Omega.$$  

The following generalization of the Shannon-McMillan theorem may be found in [4] and [8]: Let $G = \mathbb{Z}^k$, where $\mathbb{Z}$ is the group of integers and $k$ is a positive integer. For $n = 1, 2, \cdots$, let $A_n = \{(x_1, x_2, \cdots, x_k) \in \mathbb{Z}^k: 0 \leq x_i \leq n, i = 1, 2, \cdots, k\}$. Then $\{|A_n|^{-1}h_P(A_n)\}$ converges in $L^1(\Omega)$ as $n \rightarrow \infty$.

In [7] it is shown that if $G$ is the group of dyadic rationals modulo one, and if $A_n$ is the cyclic subgroup of $G$ generated by $2^{-n}$, then $\{|A_n|^{-1}h_P(A_n)\}$ converges in $L^1(\Omega)$ as $n \rightarrow \infty$. The authors of [7] conjectured that this property generalizes to a general countable abelian group.
It is the purpose of this note to announce the following theorem which generalizes these results, and settles the above conjecture. (The proofs of Theorems 1–4 will appear elsewhere.) Following [7], we call Theorem 1 the entropy equidistribution property of a measurable partition under the action of an amenable group.

**Theorem 1.** Let $K$ be a subgroup of the amenable group $G$. There exists a $K$-invariant function $h(P, T, K) \in L^1(\Omega)$ such that for every net $\{A_\alpha\}$ satisfying property $P$ with respect to $K$, $\lim_{\alpha} |A_\alpha|^{-1} h_P(A_\alpha) = h(P, T, K)$ in $L^1(\Omega)$.

The main tool used in proving Theorem 1 is the following generalized ergodic theorem which appears in [1]: If $K$ is a subgroup of $G$, $\{A_\alpha\}$ is a net satisfying property $P$ with respect to $K$, and $f \in L^1(\Omega)$, then $\{A_\alpha|^{-1} \sum_{\alpha \in A_\alpha} f \cdot T^g\}$ has a limit in $L^1(\Omega)$ which is $K$-invariant.

Define $H(P, T, K) = \int h(P, T, K) d\lambda$. Define $C(K) = \{M \in \mathcal{M} : \lambda[T^g(M) \Delta M] = 0, g \in K\}$.

**Theorem 2.** If $K_1$ and $K_2$ are subgroups of $G$ such that $K_1 \subset K_2$, then $H(K_2) \leq H(K_1)$. Equality holds if and only if $E[h(P, T, K_1) | C(K_2)] = h(P, T, K_2)$.

**Theorem 3.** If $K$ is a subgroup of $G$, there exists a countable subgroup $L$ of $K$ such that if $L'$ is any subgroup satisfying $L \subset L' \subset K$, then $h(P, T, L') = h(P, T, K)$.

**Theorem 4.** Let $K$ be a subgroup of $G$. Let $K$ be a family of subgroups of $K$ which is directed by inclusion ($\supset$), and whose union is $K$. Then $\lim_{L \in K} h(P, T, L) = h(P, T, K)$ in $L^1(\Omega)$, and $h(P, T, K) = \inf_{L \in K} h(P, T, L)$.

As an application of the foregoing results, we can define the entropy $H(T)$ of the action $T$ of the amenable group $G$ on $\Omega$ as follows: $H(T) = \sup_P h(P, T, G)$, where the supremum is over all measurable partitions $P$ of $\Omega$ with finite entropy. This definition extends that given in [2] for $G = \mathbb{Z}^k$. The entropy as we have defined it is an invariant under isomorphism. Conversely, it may be possible to generalize Ornstein's results [6] and show that generalized Bernoulli schemes (see [5] for definition) with the same entropy are isomorphic. The entropy equidistribution property (Theorem 1 above) might serve as a basic tool for proving this.
REFERENCES


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