Consider the retarded functional differential equation (RFDE)

\[ \dot{x}(t) = f(x_t) \]

where as in [1], \( x_t(\theta) = x(t + \theta), -1 \leq \theta \leq 0, x_t \in C = C([-1, 0], \mathbb{R}^n) \), and 
\[ f \in X = C^\infty(C, \mathbb{R}^n). \] Oliva [5] showed fixed points of (1) generically are hyperbolic; here we generalize to the theorem of Kupka [2], Markus [3] and Smale [7]. With an appropriate Whitney (Baire) topology on \( X \), we have

\textbf{Theorem 1.} The set of \( f \in X \) for which
1. all fixed points and all periodic solutions of (1) are hyperbolic,
2. all global unstable manifolds are injectively immersed in \( C \), and
3. all global unstable and local stable manifolds intersect transversally

is a residual subset of \( X \).

The restriction to the local stable manifold in 3. is necessary as we lack backwards existence and uniqueness.

If we consider only equations

\[ \dot{x}(t) = F(x(t - \tau_1), x(t - \tau_2), \ldots, x(t - \tau_p)), \]

(2)

with \( F \in C^\infty(\mathbb{R}^{n^p}, \mathbb{R}^n) \), we obtain

\textbf{Theorem 2.} Let \( \tau_1 = 0 \). Then the set of \( F \in C^\infty(\mathbb{R}^{n^p}, \mathbb{R}^n) \) for which
1., 2. and 3. above hold is a residual set.

The question of what happens when \( \tau_1 > 0 \) seems to be open.


\textit{Key words and phrases.} Delay differential equation, functional differential equation, generic, hyperbolic periodic orbit, stable (unstable) manifold, transversality.

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To prove Theorem 1, the various perturbations are constructed using the map \( \delta_N : C \to \mathbb{R}^{n(N+1)} \)

\[ \delta_N \phi = (\phi(0), \phi(-1/N), \phi(-2/N), \ldots, \phi(-1)). \]

The next lemma, which is easily proved, is used when \( M \) is a periodic orbit or part of an unstable manifold.

**Lemma 1.** Let \( M \subseteq C \) be an embedded \( C^\infty \) finite dimensional submanifold, and \( K \subseteq M \) compact. Then there is a neighborhood \( K \subseteq V \subseteq M \) in \( M \) such that for large \( N \), \( \delta_N : V \to \mathbb{R}^{n(N+1)} \) embeds \( V \) into \( \mathbb{R}^{n(N+1)} \) and is an immersion (and hence a diffeomorphism).

We cannot use \( \delta_N \) in Theorem 2 as all perturbations must take place in \( \mathbb{R}^{np} \). However, after approximating \( F \) with an analytic function, the following is used, with \( x(t) \) periodic or lying on an unstable manifold.

**Lemma 2.** Suppose \( \tau_1 = 0 \) and let \( x(t) \) be a solution of (2) on \( (-\infty, 0] \), where \( F \) is analytic and \( x \) is bounded on \( (-\infty, 0] \). Set \( y(t) = (x(t - \tau_1), x(t - \tau_2), \ldots, x(t - \tau_p)) \). Then

1. if \( x \) has least period \( T > 0 \), then with the exception of finitely many points in \([0, T] \), \( y \) is one-to-one on this interval;
2. if \( x \) is not periodic or constant, and \([a, b] \subseteq (-\infty, 0] \), then the same conclusion about \( y \) holds on \([a, b] \).

**Proof.** By a theorem of Nussbaum [4], \( x(t) \) is analytic, so any self-intersection of \( t \to y(t) \) in \( \mathbb{R}^{np} \) is either isolated or forms an analytic arc. In the latter case

\[ y(t) = y(\sigma(t)) \quad \text{so} \quad x(t) = x(\sigma(t)) \]

for some analytic \( \sigma \) defined in a \( t \)-interval \( I \), with \( \dot{\sigma}(t) \neq 0 \) and \( \sigma(t) \neq t \). Thus

\[ \dot{x}(t) = F(y(t)) = F(y(\sigma(t))) = \dot{x}(\sigma(t)). \]

But differentiating (3) gives \( \dot{x}(t) = \dot{x}(\sigma(t))\dot{\sigma}(t) \), so \( \dot{\sigma}(t) \equiv 1 \). Hence for some \( A \), \( x(t) = x(t + A) \) in \( I \) and thus for all \( t \) by analyticity. Thus 1 holds with \( A \) a multiple of \( T \), proving the lemma.

**References**

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THE SELBERG TRACE FORMULA FOR CONGRUENCE SUBGROUPS

BY DENNIS A. HEJHAL

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1. Introduction. The Selberg trace formula for \( SL(2, \mathbb{R}) \) is commonly understood to be a non-Abelian analog of the Poisson summation formula. The formula arises from letting a Fuchsian group \( \Gamma \) act on the upper half-plane \( \mathbb{H} \) and contains four basic contributions: identity, hyperbolic, elliptic, and parabolic [2, pp. 95–108], [3, pp. 72–79]. Because of its possible number-theoretic applications, it seems only natural to calculate the trace formula explicitly for various congruence subgroups of \( SL(2, \mathbb{Z}) \) and see what happens.

From the general theory, one knows that the parabolic (or arithmetic) contribution will be \( \text{Tr}(M) \), where

\[
M = \frac{1}{4\pi} \int_{-\infty}^{\infty} h(r) \phi'(s) \phi(s)^{-1} \, dr + \frac{1}{4}[I - \phi(\frac{1}{2})] h(0) \\
- \left[ g(0) \ln 2 + \frac{1}{2\pi} \int_{-\infty}^{\infty} h(r) \frac{\Gamma'(1 + ir)}{\Gamma(1 + ir)} \, dr \right] I.
\]

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