A GENERALIZED WEIL TYPE REPRESENTATION AND A FUNCTION ANALOGOUS TO $e^{-x^2}$

BY TOMIO KUBOTA

Communicated by D. J. Lewis, April 28, 1975

Let $dV(z)$ be the euclidean measure of $C$, and let $n \geq 2$ be a natural number. Put $e(z) = \exp(\pi \sqrt{-1}(z + \bar{z}))$, $\zeta = e(1/n)$, and consider the Hilbert space $H_n$ consisting of all functions $\Phi$ on $C$ such that $\Phi(\zeta z) = \Phi(z)$ and $\|\Phi\| < \infty$, where the norm is coming from the inner product

$$(f_1, f_2) = \int_C f_1(z) \overline{f_2(z)} |z|^{2n-4} dV(z).$$

Denote by $\Phi \rightarrow \Phi^*$ the integral linear transformation given by

$$\Phi^*(t) = \int_C \Phi(z) k(zt) |z|^{2n-4} dV(z)$$

with $k(z) = n^2 \lim_{Y \to \infty} \int_{|z| < Y} e(z^n/\omega^n) e(\omega^n) dV(z)$.

Denote furthermore by $\sigma = (a, b; c, d)$ an element of $G = SL(2, C)$ for which $(a, b)$ is the first row and $(c, d)$ is the second, and define an operator $r_n(\sigma)$ of $H_n$ for three types of elements $\sigma_1 = (a, 0; 0, a^{-1})$, $\sigma_2 = (1, b; 0, 1)$, and $\sigma_3 = (-c^{-1}, 0; c, 0)$ by

$$(r_n(\sigma_1) \Phi)(t) = |a|^{n(n-1)/2} \Phi(a^{2/n} t),$$

$$(r_n(\sigma_2) \Phi)(t) = |c|^{-n(n-1)/2} \Phi^*(c^{-2/n} t).$$

Then, it follows from the results, to be announced in [2] in detail, that $r_n(\sigma)$ extends multiplicatively to an irreducible unitary representation $\sigma \rightarrow r_n(\sigma)$ of $G$ of class one on $H_n$ belonging to the supplementary series. If $n = 2$, then $k(z)$ reduces to $e(2z) + e(-2z)$, and $\sigma \rightarrow r_n(\sigma)$ reduces essentially to a special case of the representation given in [3].

These results, viewed so to speak from the reverse side, yield as a byproduct a representation theoretic characterization of a special function. Namely, we obtain

**Theorem.** Up to a constant factor, the function $h(t) = t K_{1/n}(2n|t|^n)$ is the only function in $H_n$ which is invariant by all $r_n(\sigma)$ with $\sigma \in K = SU(2)$, where $K_{1/n}$ is a modified Bessel function.

This Theorem follows from the facts, proved in [2], that $h(t)$ is actually invariant by all $r_n(\sigma)$, $(\sigma \in K)$, and that the set of all $r_n(\sigma) h(t)$, $(\sigma \in G)$, is dense in $H_n$.
If $n = 2$, then $h(t)$ reduces to $\frac{1}{2}e^{-2\pi |x|^2}$, so that the above theorem with $n = 2$ is easily derived from, and is practically equivalent to, a special case of the result in [1, Chapter 1, §8, Theorem 9], which gives a conceptual characterization of functions of the type $e^{-x^2}$.

REFERENCES

