I consider the following parabolic equation

\[ u_t = \text{div} \ A(x, t, u, u_x) + B(x, t, u, u_x) \tag{1} \]

where \( A, B \) are respectively, vector and scalar valued measurable functions satisfying the structure conditions

\[
|A(x, t, u, p)| \leq a_1 |p| + a_2 |u| + a_3, \quad |B(x, t, u, p)| \leq b_1 |p| + b_2 |u| + b_3,
\]

(2)

\[ p \cdot A(x, t, u, p) \geq c_1 |p|^2 - c_2 |u|^2 - c_3, \]

where \( a_1, c_1 \) are positive constants, and all of the remaining coefficients \( a_p, b_p, c_i \) are in \( L^{p,q} \) for some pair of numbers \( (p, q) \) satisfying \( p > 2/(1 - \theta) \); \( n/p + 2/q \leq 1 - \theta \), where \( \theta \) is a positive constant, \( 0 < \theta < 1 \). This is precisely the equation studied by Aronson and Serrin [1] and is very similar to that studied by Trudinger [7].

We consider weak solutions from the class \( V^2 \) in cylinders \( Q = \Omega \times (0, T) \) where \( \Omega \subset \mathbb{R}^n \) is a bounded domain. \( V^2(Q) \) is defined to be the space of measurable functions \( u \) which have finite norm

\[
\|u\|_{V^2(Q)} = \text{ess sup} \sup_{0<t<T} \left\{ \int_{\Omega} |u(x, t)|^2 \, dx \right\}^{1/2} + \sum_{i=1}^n \|\frac{\partial u}{\partial x_i}\|_{L^2(Q)}
\]

where \( \{\partial u/\partial x_i\}_{i=1,...,n} \) are the weak (i.e. distributional) derivatives of \( u \). We define \( V^2_0(Q) \) to be the closure in \( \| \cdot \|_{V^2(Q)} \) of functions in \( C^\infty(Q) \) which vanish in a neighborhood of the parabolic boundary \( \partial_p Q = \bar{\Omega} \cup \{\partial \Omega \times [0, T]\} \).

We say that \( u \in V^2_0(Q) \) is a weak solution to (1) if \( \int \varphi_t u - \varphi_x \cdot A(x, t, u, u_x) + \varphi B(x, t, u, u_x) = 0 \) for every function \( \varphi \in C^\infty_c(Q) \).

The Maximum Principle for such equations (Aronson and Serrin [1, Theorem 1], can be generalized to the notion of weak boundary values as follows.

**Theorem.** If \( u \in V^2_0(Q) \) is a weak solution to (1) then almost everywhere in \( Q \) we have

\[ |u(x, t)| \leq C(\|b_3\|_{p,q}^2 + \|c_3\|_{p,q}) \]
where \( C = C(T, |\Omega|, n, \theta, \|b_1\|, \|b_2\|, c_1, \|c_2\|) \).

We employ the familiar Bessel capacity \( B_{1,2} \) on \( \mathbb{R}^n \) (see Meyers [5]) and introduce a new capacity \( VC \) defined on \( \mathbb{R}^{n+1} \) by

\[
VC(A) = \inf \{ \|u\| \sqrt{2} \mathbb{R}^{n+1} : u \geq 0 \text{ and } A \subset \text{Int}\{(x, t): u(x, t) \geq 1\} \}
\]

for any set \( A \subset \mathbb{R}^{n+1} \). \( VC \) is an outer measure on \( \mathbb{R}^{n+1} \). These capacities are employed in the following results.

**Theorem.** If \( u \in V_0^2(Q) \) is a weak solution of (1), then

\[
\lim_{(x,t) \to (x_0, t_0) ; (x,t) \in Q} u(x, \ t) = 0
\]

for \( VC \) almost every point \((x_0, t_0) \in \partial \Omega \).

**Theorem.** Suppose \( n > 2 \) and the structure coefficients \( a_p, b_p, c_i \) in (2) are all positive constants. Suppose also that \( x_0 \in \partial \Omega \) has the property that the \( B_{1,2} \) upper capacitary density of \( \tilde{\Omega} \) is positive at \( x_0 \), that is

\[
\limsup_{r \to 0} \frac{B_{1,2}(B(x_0, r) \cap \tilde{\Omega})}{B_{1,2}(B(x_0, r))} > 0.
\]

If \( u \in V_0^2(Q) \) is a weak solution of (1), then \( \lim_{(x,t) \to (x_0, t_0); (x,t) \in Q} u(x, \ t) = 0 \) for every \( t_0 \in (0, T) \).

The notion of limit employed is, of course, the essential limit. That is, \( u \) may need to be redefined on a set of zero measure.

The last result is modeled after a similar result for elliptic equations appearing in [3]. It gives a Weiner-like geometric condition on the base region \( \Omega \) which implies continuity of a weak solution at all points on the lateral boundary of the cylinder directly "above" the boundary point \( x_0 \). Proofs of all results appear in [2].

**BIBLIOGRAPHY**


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