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Partial differential equations, by E. T. Copson, Cambridge University Press, Cambridge (England), London, New York, Melbourne, 1975, vii + 280 pp.

Many basic laws of nature can be formulated as systems of differential equations, ordinary or partial. Predictions of physical phenomena then present themselves as boundary problems for such systems. Many of them are formidable mathematical challenges not yet mastered. Those which have been solved have required the entire arsenal of analysis, power series, separation of variables, successive approximations, Fourier analysis, functional analysis and distributions. On the other hand, most of these tools were created to solve problems in physics. The classical linear partial differential equations are Laplace's equation of potential theory, the wave equation of the theory of wave propagation, and the heat equation of the theory of heat conduction. The diversity of the physics involved explains the fact that the corresponding boundary problems are quite different and also the methods for their solution.

Riemann's lectures, *Partial differential equations and their applications*, published by Hallendorff in 1882, was the first systematic book in the field. Twenty years later came an expanded version by Weber, which after another twenty years branched out into the encyclopedic *Differential equations of physics* by Frank and von Mises. At about the same time, *Methods of mathematical physics* by Courant and Hilbert, and Webster's *Partial differential equations of mathematical physics* made their appearance. The aim was twofold: to give old and new mathematical tools to physicists and to introduce mathematicians to some interesting physics. The men who wrote these books were mathematicians fascinated by the richness and variety of classical physics. Webster opens his book with the following sentence: "It is the lofty aim of mathematical or theoretical physics to describe the universe." Being a mathematician he also wanted economy of description and, if possible, unity of method. Courant had the same attitude. After commenting in his preface on the threatening divorce of mathematics and physics brought about by increasing specialization, he expresses hope that his book will give the reader access to a rich and important field held together by far-reaching mathematical theories.

The time when these major works were published also saw the birth of quantum mechanics. In this new branch of physics, algebra, groups and operator theory were quite as important as partial differential equations. Further development of this old field was in fact left to the mathematicians. Functional analysis in the shape of fixed-point theorems contributed existence theorems in nonlinear situations, and linear functional analysis began slowly to penetrate the subject. Since mathematicians are never content with nature, they also tried to extend the theory of the three classical equations, all of order 2, to higher order equations. Major contributions by Petrovsky in the thirties laid the foundations of a general theory of elliptic, hyperbolic and parabolic systems of equations. In the fifties, Ehrenpreis, Hörmander and Malgrange developed the beginnings of a general theory of linear partial differential operators. They used the theory of distributions by Schwartz which gave them a new freedom in the handling of the Fourier transform and the treatment of singularities. Without this theory, recent developments like pseudodifferential operators and microlocal analysis are unthinkable.

Partial differential equations used to be just a convenient label for a set of somewhat loosely connected methods and results but these rapid and partly revolutionary developments have given it the status of a field of mathematics independent of physics.

There are at present plenty of modern books covering various aspects of partial differential equations but the day of comprehensive treatments has probably passed. Increased interest in the field has also created a demand for elementary introductions. Every such text has to adjust to its prospective readers and to the preferences of its author.[1] Copson wrote his text because he found nothing really suitable for beginners and wanted to give them something in the classical tradition. His book is dedicated to the memory of Sir Edmund Whittaker. There are four major themes: equations of the first order; hyperbolic equations in two, three and four variables, including Riesz's fractional integrals; elliptic equations with potential theory in the plane and in space, including the exterior Dirichlet problem and Sommerfeld's radiation condition; and, finally, heat conduction. Every chapter closes with a collection of exercises. The text is carefully written and the choice of material is natural. Emphasis and terminology are on the conservative side. It would have been possible to take some of the modern development into account without

[1] But since there is considerable latitude in both these variables, the chances are very small that a book of this kind will be universally accepted.

changing the level of the book, but I do not want to go into the details of this. Instead I should like to make a remark that may be of general interest.

Copson's book is full of examples from physics but they are not presented in a very systematic way. Sometimes they just illustrate the algebraic classification into types, sometimes they motivate the classification. I believe that physics should have the lead and this for two reasons. A full physical introduction to, e.g., wave propagation gives the reader in one stroke many intuitive aspects that will appear later in the mathematical theory. Also, when the physicists turned to quantum mechanics, they left classical physics, heat, electricity, hydrodynamics and waves to the engineers, who, unfortunately, have to specialize in one of these branches. The mathematicians are the only ones who now have the opportunity of giving students something like a general education in classical physics. This opportunity should be used. Wave propagation, potential theory and heat conduction should appear as early as reasonably possible in every standard calculus course. The same goes for that indispensable tool, the Fourier transform. Neglecting these simple applications, calculus is not really a serious affair and–to use Webster's words–its lofty aim may get out of sight.

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Elliptic functions, by Serge Lang, Addison Wesley, Reading, Mass., 1973, xii + 326 pp., $17.50.

Elliptic functions have attracted and fascinated many generations of mathematicians. For more than 150 years, these functions have kept their place in the center of mathematical interest and activity. Their appeal to us can perhaps be explained by their structural universality, what the author calls the intermingling of Analysis, Algebra and Arithmetic (the 3 Gaussian A's)– and thus a sizeable portion of Mathematics. In view of this, the theory of elliptic functions is considered to be a "deep" theory. Moreover, elliptic functions are the first nontrivial examples of the more general abelian functions. Not only do general theorems about abelian functions become explicit and more lucid in the case of elliptic functions, but also do special results about elliptic functions often constitute the first stepping stone on the way to their generalization in the abelian case.

Treatises on elliptic functions are numerous and it seems futile to attempt a classification. As to the book under review, its place in the existing literature is perhaps best described by saying that it continues the tradition of the classics by Weber and Fricke, including also more recent results which are connected with the names of Hasse, Deuring and Shimura. Specifically, a large part of the book is devoted to the body of results known under the name of "complex multiplication". These results are concerned with the so-called singular values of elliptic modular functions, and it is shown that they serve to generate abelian extensions of imaginary quadratic number fields. In addition to complex multiplication, the author also considers the case of nonsingular values of modular functions and the fields they generate. This includes the