Quillen’s recent solution [4] of Serre’s problem (on projective modules over polynomial rings) is based on the following remarkable theorem.

Let \( K \) be a commutative ring, \( \text{max}(K) \) its set of maximal ideals, and \( T \) an indeterminate.

**Theorem 1** (Quillen [4, Theorem 1]). Let \( M \) be a finitely presented \( K[T] \)-module and put \( M_0 = M/\text{T}M \). If \( M_m \cong M_0[T]_m \) for all \( m \in \text{max}(K) \), then \( M \cong M_0[T] \).

We have developed an axiomatic version of Quillen’s arguments, also using ideas of [1], which yields the following results, among others. Detailed proofs will appear elsewhere.

**Theorem 2.** Theorem 1 is valid with the word “module” replaced by “algebra”.

Theorem 1 follows from Theorem 2, applied to the symmetric algebra \( S(M) \).

Call a commutative \( K \)-algebra \( A \) invertible if, for some \( K \)-algebra \( B \), \( A \otimes_K B \) is a polynomial algebra \( K[X_1, \ldots, X_n] \). Then \( A \) admits an augmentation, \( 0 \rightarrow \overline{A} \rightarrow A \rightarrow K \rightarrow 0 \), and the \( K \)-module \( JA = \overline{A}/\overline{A}^2 \) depends, up to isomorphism, only on \( A \). We say \( A \) is stably isomorphic to a \( K \)-algebra \( B \) if \( A \otimes_K C \cong B \otimes_K C \) for some invertible \( K \)-algebra \( C \).

**Theorem 3.** Let \( A \) be a finitely presented \( K \)-algebra.

(a) If \( A_m \) is a polynomial \( K_m \)-algebra for all \( m \in \text{max}(K) \) then \( A \) is a symmetric algebra \( S(P) \) of a projective \( K \)-module \( P \).

(b) If \( A_m \) is an invertible \( K_m \)-algebra for all \( m \in \text{max}(K) \) then \( A \) is invertible.

**Corollary.** Let \( A \) and \( B \) be invertible \( K \)-algebras. If \( JA \) and \( JB \) are stably isomorphic, and if \( A_m \) and \( B_m \) are stably isomorphic for all \( m \in \text{max}(K) \), then \( A \) and \( B \) are stably isomorphic.

**Remarks.** The title of the paper refers to part (a), which solves a problem posed in [2, p. 67], [3], [5, §6], and [6, p. 3]. In geometric language it asserts that every affine space bundle over \( \text{spec}(K) \) arises from a vector bundle. Part (b)
is proved by reducing it to part (a). Part (a) is proved by first constructing an augmentation \( A \to K \) and then applying the following general result, which has various other applications.

**Theorem 4.** Let \( A \) be a finitely presented (not necessarily commutative) \( K \)-algebra equipped with an augmentation, \( 0 \to \widetilde{A} \to A \to K \to 0 \), and put \( \gamma A = \bigoplus_{n \geq 0} \widetilde{A}^n / \widetilde{A}^{n+1} \), the associated graded algebra. If \( A_n \cong \gamma A_n \) (as filtered algebras) for all \( m \in \text{max}(K) \), then \( A \cong \gamma A \).

**References**