1. **Introduction.** In this note we determine the bordism groups $A_n$ of orientation preserving diffeomorphisms of $n$-dimensional closed oriented smooth manifolds. These groups were introduced by W. Browder [1]. Winkelnkemper showed that each diffeomorphism of the sphere $S^n$ is nullbordant [7]. On the other hand, he showed that $A_{4k+2}$ is not finitely generated. Medrano generalized this result to $A_{4k}$ [5]. For this he introduced a powerful invariant in the Witt group $W_\pm(Z, Z) (I_\pm$ in Medrano’s notation) of isometries of free finite-dimensional $Z$-modules with a symmetric (antisymmetric) unimodular bilinear form. The invariant is given by the middle homology modulo torsion, the intersection form and the isometry induced by the diffeomorphism. For a diffeomorphism $f: M \to M$ we denote this invariant by $I(M, f)$, the isometric structure of $(M, f)$. It is a bordism invariant and leads to a homomorphism $I: A_{2k} \to W_{(-1)k}(Z, Z)$.

Neumann has shown that the homomorphism $I$ is surjective, that $W_+(Z, Z) \otimes Q \cong Q^\infty$ and that $W_\pm(Z, Z)$ contains infinitely many summands of orders 2 and 4 [6]. On the other hand, $W_\pm(Z, Z)$ is a subgroup of $W_\pm(Z, Q)$, the Witt group of isometries of finite-dimensional $Q$-vector spaces. This group plays an important role in the computation of bordism groups $C_{2k-1}$ of odd-dimensional knots, which can be embedded in $W_{(-1)k}(Z, Q)$. It is known that $W_\pm(Z, Q) \cong Z^\infty \oplus Z_2^\infty \oplus Z_4^\infty$ [3]. Thus the group $W_\pm(Z, Z)$ is also of the form $Z^\infty \oplus Z_2^\infty \oplus Z_4^\infty$.

It turns out that the isometric structure is essentially the only invariant for bordism of diffeomorphisms.

2. **Bordism of odd-dimensional diffeomorphisms.** Two diffeomorphisms $(M_1, f_1)$ and $(M_2, f_2)$ are called bordant if there is a diffeomorphism $(N, F)$ on an oriented manifold with boundary such that $\partial(N, F) = (M_1, f_1) + (-M_2, f_2)$. The bordism classes $[M', f]$ form a group under disjoint sum, called $\Delta_n$.

The mapping torus of a diffeomorphism $(M, f)$ is $M_f = I \times M/(0, x) \sim (1, f(x))$. This construction leads to a homomorphism $\Delta_n \to \Omega_{n+1} ([M, f] \mapsto [M_f])$, where $\Omega_{n+1}$ is the ordinary bordism group of oriented manifolds.

In [4] we proved the following result.

---

*AMS (MOS) subject classifications (1970).* Primary 57D90.
**Theorem 1.** For $k > 2$ the map $[M, f] \mapsto ([M], [M_f])$ is an isomorphism $\Delta_{2k-1} \rightarrow \Omega_{2k-1} \oplus \hat{\Omega}_{2k}$, where $\hat{\Omega}_{2k}$ is the kernel of the signature homomorphism $\tau$.

3. **The even-dimensional case.** Consider triples $(G, \langle , \rangle, h)$, where $G$ is a finite-dimensional free $\mathbb{Z}$-module, $\langle , \rangle$ a symmetric (resp. antisymmetric) unimodular bilinear form on $G$ and $h$ an isometry of $(G, \langle , \rangle)$. $(G, \langle , \rangle, h)$ is called hyperbolic if there exists an invariant subkernel, i.e. a subspace $U \subset G$ with $U \subset U_1$, $2 \dim U = \dim G$ and $h(U) \subset U$. $(G, \langle , \rangle, h)$ and $(G', \langle , \rangle', h')$ are called bordant if $(G, \langle , \rangle, h) \oplus (G', -\langle , \rangle', h')$ is hyperbolic. This is an equivalence relation. The equivalence classes form a group under orthogonal sum, called $W_+ (\mathbb{Z}, \mathbb{Z})$ (resp. $W_- (\mathbb{Z}, \mathbb{Z})$).

The isometric structure of a diffeomorphism $(M^{2k}, f)$ is given by $(H_k (M; \mathbb{Z})/\text{Tor}, \circ, f_\ast)$, where $\circ$ is the intersection form. If $(M, f)$ bounds a diffeomorphism $(N, F)$ the isometric structure is hyperbolic, an invariant subkernel being given by the kernel of $i_\ast: H_k (M; \mathbb{Z})/\text{Tor} \rightarrow H_k (N; \mathbb{Z})/\text{Tor}$, so we have a homomorphism $I: \Delta_{2k} \rightarrow W_{(-1)k} (\mathbb{Z}, \mathbb{Z})$. Neumann has shown that this homomorphism is surjective [6].

**Theorem 2.** For $k > 1$ the homomorphism

$$
\Delta_{2k} \rightarrow W_{(-1)k} (\mathbb{Z}, \mathbb{Z}) \oplus \hat{\Omega}_{2k} \oplus \Omega_{2k+1},
$$

$$
[M, f] \mapsto (I(M, f), [M] - \tau(M) [P_k C], [M_f])
$$

is an isomorphism ($k$ even), injective with cokernel $\mathbb{Z}_2 (k$ odd).

4. **Idea of the proof.** Consider a diffeomorphism $(M^n, f)$ such that $M$ and $M_f$ are nullbordant. This implies that there exists a differentiable map $g: N^{n+2} \rightarrow S^1$ with $\partial N = M_f$ and $g|_{\partial N}$ the canonical projection from $M_f$ to $S^1$. Let $x \in S^1$ be a regular value of $g$. $F^1 = g^{-1}(x)$ is a 1-codimensional submanifold of $N$ with trivial normal bundle meeting $\partial N$ transversally along $\partial F$: Cut $N$ along $F$ to obtain a differentiable manifold $N_F$ with corners. The boundary of $N_F$ consists of two copies $F_0$ and $F_1$ of $F$ with opposite orientations and a manifold $V = \partial N_{\partial F}$ fibred over the unit interval $I$. $\partial V = \partial F_0 + \partial F_1$. The corners of $N_F$ are at $\partial F_0$ and $\partial F_1$.

We now make the following strong assumption (compare [2, 2.3]): The components of $F$ are simply connected and $F_0$ and $F_1$ are deformation retracts of $N_F$.

Then $(N_F; F_0, F_1)$ is a relative $h$-cobordism between $(F_0, \partial F_0)$ and $(F_1, \partial F_1)$. For $n > 5$ the $h$-cobordism theorem implies that the diffeomorphism on $M$ can be extended to a diffeomorphism of $F$.

Our aim is, starting with an arbitrary $N_F$ to obtain an $N_F'$, which satisfies the above assumption. For this we modify $N_F$ by addition and subtraction of handles. If $n$ is odd ($n \neq 3$) we have shown in [4] that this works. If $n$ is even
(n > 2) and \( I(M, f) = 0 \) we can do the same. The details of the proof will appear elsewhere.

REFERENCES


MATHEMATISCHES INSTITUT DER UNIVERSITÄT, SONDERFORSCHUNGSBE-REICH THEORETISCHE MATHEMATIK, WEGELERSTR. 10, 53 BONN, GERMANY