In this paper $G$ is a compact abelian group with ordered dual $\Gamma$. By this we mean there is a nontrivial group homomorphism $\phi: \Gamma \rightarrow \mathbb{R}$ where $\mathbb{R}$ is the additive group of real numbers. Let $M(G)$ be the usual convolution algebra of finite Borel measures on $G$ and $^\wedge$ the Fourier-Stieltjes transformation.

A measure $\mu \in M(G)$ is said to vanish at infinity in the direction of $\phi$ if $\{\gamma_n\}_\infty^\infty \subset \Gamma$ with $\phi(\gamma_n) \rightarrow \infty \Rightarrow \hat{\mu}(\gamma_n) \rightarrow 0$. The subspace consisting of all measures whose transforms vanish at infinity in the direction of $\phi$ will be denoted by $M_0(G)$.

Let $\delta_0$ be the identity measure in $M(G)$ and for any integer $N_i$ put $\delta_i = N_i \delta_0$. The purpose of this note is to announce the following results which explicate a line of research begun by H. Helson [2] and continued by various authors in [1], [3], [5], [6], and [7].

**Theorem 1.** Let $\mu \in M(G)$ such that the convolution product $\prod_{i=1}^m (\mu - \delta_i) \in M_0(G)$. Then $\mu$ has a decomposition $\mu = \mu_0 + \mu_\perp$ where $\mu_0 \in M_0(G)$, $\mu_\perp \in M_\perp(G)$ and $\hat{\mu}_\perp(\Gamma) \subset \{N_1, \ldots, N_m\}$. If $\prod_{i=1}^m (\mu - \delta_i) \in M_0(G)$ then $\mu$ has a decomposition $\mu = \mu_0 + \mu_\perp$ where $\mu_0 \in M_0(G)$, $\mu_\perp \in M_\perp(G)$ and $\hat{\mu}_\perp(\Gamma) \subset \{N_1, \ldots, N_m\}$. Here $M_0(G)$ is the ideal of measures $\mu \in M(G)$ such that $\hat{\mu} \in C_0(\Gamma)$.

The proof of Theorem 1 is obtained by analyzing $\mu_\perp$ in $M(S)$ where $S$ is the structure semigroup of $M(G)$.

Assume $\phi$ is an isomorphism, $P$ the positive cone and $E$ a Sidon subset of $\Gamma$. For any subset $A$ of $\Gamma$ put $F(A) = \{\mu \in M(G): \hat{\mu}$ is integer-valued on $A \}$ and $I(A) = \{\mu \in M(G): \hat{\mu} = 0$ or 1 on $A \}$. The following theorem is a consequence of Theorem 1 and is an extension of a result announced by I. Kessler [3]; see also [4, pp. 206–211].

**Theorem 2.** If $\mu \in F(\Gamma \setminus P \cup E)$ then there is a $\nu \in F(\Gamma)$ such that $\hat{\mu} = \hat{\nu}$ off $- P \cup E$. In particular, if $\mu \in I(\Gamma \setminus P \cup E)$ then $\nu \in I(\Gamma)$.

Measures such that $\hat{\mu}(\gamma) = \hat{\nu}^2(\gamma)$ for all $\gamma \in P$ are called semi-idempotents. A subset $R$ of $\Gamma$ is said to be a weak Rajchman set if supp $\hat{\mu} \subset R \Rightarrow \hat{\mu} \in C_0(\Gamma)$. An easy consequence of Theorem 1 is the following result.
THEOREM 3. If $\mu \in F(\Gamma \setminus \mathbb{R})$ then there is a $\nu \in F(\Gamma)$ such that $\hat{\mu} = \hat{\nu}$ off $\mathbb{R}$. In particular, if $\mu \in I(\Gamma \setminus \mathbb{R})$ then $\nu \in I(\Gamma)$.

For examples of Rajchman sets, the reader is referred to [5]. Proofs of our results will appear elsewhere.

REFERENCES


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