naturally. They are the new lines replacing each P_i (6 of these), the image of lines in $\mathbf{P}^2$ passing through two of the P_i (15 of these), and the image of conics in $\mathbf{P}^2$ passing through all but one of the P_i (6 of these). This story and the background material that goes into it form a marvelous chapter in the theory of algebraic surfaces.

I hope these three examples will give some idea of what this book is like. It is full of ideas ranging over a wide area but always centered around algebraic varieties in complex projective space. It is not a systematic introduction to algebraic geometry. Rather it is a sampler of a number of methods and results proved by whatever techniques of topology, differential geometry, complex analytic geometry, or commutative algebra lie closest at hand, which should stimulate the interest and imagination of any reader.

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Semisimple Lie algebras, by Morikuni Goto and Frank D. Grosshans, Lecture Notes in Pure and Applied Mathematics, No. 38, Marcel Dekker, Inc., New York, 1978, vii + 480 pp., $37.50.

Lie groups pose a problem for both the learner and the teacher (or textbook writer). Topology, analysis, algebra are so intertwined that no expository scheme can do full justice to the subject without becoming encyclopedic. On the other hand, this diversity of aspect, coupled with a wide range of applicability, makes Lie theory especially attractive.

There are now available a substantial number of books dealing with semisimple Lie groups and/or Lie algebras. These differ considerably in scope and emphasis, but most are built around a common core of Lie algebra theory: nondegeneracy of the Killing form, root system and Weyl group, classification (over $\mathbf{C}$ and perhaps over $\mathbf{R}$), automorphism groups, compact real forms, Cartan decomposition of a real form, finite-dimensional representations, Weyl's character and dimension formulas. This theory reached a certain degree of completeness in the 1930s, following the fundamental work of E. Cartan and H. Weyl; later workers, notably C. Chevalley, have expanded and improved the edifice, without obliterating much of the original architecture.

In his excellent review [**4**], A. W. Knapp has pointed out some of the problems that beset all authors who write about semisimple Lie groups. One solution to the problem of where to begin is to begin with Lie algebras, thereby avoiding or at least postponing the more elaborate foundations required for the groups. This approach sacrifices the geometric motivation for studying Lie groups, but retains the common core mentioned above and makes it possible to penetrate to that core in a reasonably short time.

The book of Goto and Grosshans is intended "to give an account of that part of the theory of Lie algebras most relevant to Lie groups" and also to introduce the reader to linear Lie groups. In about 480 typewritten pages (equivalent perhaps to 250 typeset pages) they have managed to hit most of the high spots. Here is an outline of what they do.

The book begins with some generalities on Lie algebras, including a statement without proof of the Poincaré-Birkhoff-Witt theorem and a proof (following H. Zassenhaus) that every finite-dimensional Lie algebra over a field of characteristic 0 has a faithful finite-dimensional representation (this uses results from Chapters 2 and 3). Chapter 2 contains a thorough discussion of semisimple Lie algebras, leading up to the classification by Dynkin diagrams (or "π-systems"). Although the goal is "to show that the classification of simple Lie algebras over an algebraically closed field P of characteristic 0 is the same as the classification of π-systems", the authors neglect to point out that this depends on the conjugacy of all Cartan subalgebras, which is proved only later (Chapter 4), in the case $P = \mathbf{C}$. The classical Lie algebras (types A–D), and later the corresponding classical linear Lie groups, are discussed in some detail, but there is no general existence theorem for semisimple Lie algebras.

Chapter 3 introduces cohomology groups and Casimir operators, leading to the standard applications: complete reducibility of finite-dimensional representations of a semisimple Lie algebra, classification of extensions with an abelian kernel, existence and conjugacy of Levi factors.

Chapter 4 compares semisimple Lie algebras over $\mathbf{R}$ and $\mathbf{C}$: compact real form, Cartan decomposition in an arbitrary real form, Iwasawa decomposition, automorphism groups and conjugacy of Cartan subalgebras. To construct a compact real form, the authors use a Weyl basis but do not go so far as to introduce a Chevalley basis (for which the structure constants would be integral). Here and elsewhere their approach is both traditional and straightforward.

The first half of the book concludes with a nice chapter devoted to various groups associated with a semisimple Lie algebra g (or compact real form u): the Weyl group of the root system Δ, denoted $\mathrm{Ad}(\Delta)$, along with $\mathrm{Aut}(\Delta)$, and their relations with $\mathrm{Aut}(g)$, $\mathrm{Aut}(u)$; the affine Weyl group corresponding to the extended Dynkin diagram, and its relation with the universal center (this is a thorough account, following [**3**]). The study of $\mathrm{Aut}(u)$ makes essential use of a result from Chapter 6: automorphisms of u which yield inner automorphisms of g are already inner. (Such forward references make the logical structure of the book a bit more complicated than it seems at first.)

Most of the roughly 120 exercises in the book occur in these five chapters, which could serve as the basis for a one semester graduate course having only mild prerequisites (apart from affluence!). But for this purpose Samelson's notes [**5**] would be an alternative to consider.

The remainder of the book is more sophisticated and, contrary to what the authors state in their preface, far from being "fairly self-contained". This poses a serious problem for the newcomer to the subject, and makes one wonder how the authors pictured their potential reader. To quote their chapter introduction: "In 6.9, the last section of the chapter, we give H. Weyl's theorems on compact Lie groups, where we use results on general Lie groups. But the reader who knows the materials in [Chevalley] will have no trouble in understanding anything in 6.9 and in this chapter." Later (p. 308): "This section requires elementary knowledge of Lie groups. The reader can find all the background necessary in [Chevalley], and we shall not mention the references each time." This blanket reference to [**1**] is in sharp contrast with the generally elementary tone of the earlier chapters, and is even more at odds with the level of the appendix (24 pages of smaller type) titled *Notes on linear operators*; there one finds a definition of "linear transformation", a proof of the Cayley-Hamilton theorem, and other standard material. This appendix is entirely uncoordinated with the text, e.g., on p. 80 the proof of (2.1.8) is omitted, but the reader is not told to look for it in (A.3.2). It seems a bit perverse to include such an appendix while merely sketching the proof of the Poincaré-Birkhoff-Witt theorem "in order to save the space".

The first seven sections of Chapter 6 (Linear Groups) use relatively little manifold theory, since Lie groups are at first required to be closed subgroups of $\mathrm{GL}(r, \mathbf{C})$ or $\mathrm{GL}(r, \mathbf{R})$. The Lie algebra g of G is just defined to consist of the matrices X for which the 1-parameter group $\exp(\mathbf{R}X)$ lies in G. These sections treat: examples of classical linear groups and other algebraic groups, polar decomposition of $\mathrm{GL}(r, \mathbf{C})$, Cartan and Iwasawa decompositions of a semisimple linear group, construction of a linear group having prescribed Lie algebra. But then the going gets rough, with a treatment in (6.8) of the flag manifold attached to a compact semisimple Lie algebra u, notably the proof that it is simply connected (quoting both [**1**] and a technical lemma from Helgason's book). In (6.9) this is used along with other machinery (such as Peter-Weyl theory) to obtain Weyl's main theorems on compact Lie groups: compactness of the universal covering group, surjectivity of exp, etc.

Chapter 7 deals with irreducible finite-dimensional representations over $\mathbf{C}$ and then over $\mathbf{R}$. This starts with Cartan's classification by highest weight, done straightforwardly. But the existence of a representation with given highest weight and the proof of the character formula are done globally, following Weyl; so (6.9) is used heavily. This approach yields group representations and ties in nicely with the abstract discussion of the universal center in Chapter 5. But the reader has to be willing to work quite hard; for example, one proof (p. 340) concludes tersely: "By results of dimension theory, $U^{(r)} = U - U^{(s)}$ is connected and simply connected."

The comparison of irreducible representations of a real semisimple Lie algebra with those of the complexified algebra over $\mathbf{C}$ is worked out in detail, following Cartan. This comparison is somewhat tricky, though elementary, and would be easier to follow if illustrated by examples (but none are given).

Finally, Chapter 8 carries out the classification (due to Cartan, with improvements by F. Gantmacher and others) of real forms of a complex semisimple Lie algebra. The chapter begins with the determination of maximal subalgebras of maximal rank in a compact Lie algebra, following A. Borel, J. de Siebenthal. Here, as elsewhere in the book, there is some lack of connecting narrative; in particular, the reader is not told right away why these subalgebras are being investigated. Again some crucial use is made of results from (6.9), but otherwise the exposition is self-contained.

This outline indicates both some strengths and some weaknesses in the authors' approach. Some of the topics they cover (affine Weyl groups, real representations, classification of real forms) are not so often treated in other standard books, so the researcher in Lie theory may find this book quite helpful as a reference. But for Lie algebras, Samelson [5] probably provides a better first course, and for Lie groups, Varadarajan [6] tells the story in a more organized, self-contained way. (For an approach emphasizing linear Lie groups, [2] would also be well worth considering if it were written in something closer to English.) The ultimate book has not been–and doubtless will not be–written. Goto and Grosshans have given a generally clear account of semisimple Lie algebras in the spirit of Cartan and Weyl, but they have not reached a fully satisfactory compromise in their parallel treatment of Lie groups.

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Closed graph theorems and webbed spaces, by M. De Wilde, Research Notes in Mathematics, no. 19, Pitman, London, 1978, xii + 158 pp.

It was in the early thirties, during the depression, that the twins (if I may call them so) of Functional Analysis were born, thanks to Banach [1]. I mean, of course, the open mapping (o.m.) and closed graph (c.g.) theorems: Let E, F be two complete metrizable topological vector spaces (called F-spaces for short). Then (o.m.): each linear continuous map of E onto F is open; (c.g.): each linear map of F into E with closed graph is continuous. (These are two