equicoloration theorem has been shown by Hajnal and Szemeredi.

There are many other subjects covered in this volume and we shall not attempt to enumerate them here. The topics covered are generally discussed in depth. The book, though self-contained, would be difficult reading without some prior basic knowledge of Graph Theory. The pace is brisk and the reader is quickly brought to the frontiers of the subject.

Bollobás is a fastidious writer. The theorems are precisely stated and the proofs are carefully written. The publisher, Academic Press, has done a fine job. Most important, Bollobás is a mathematician who knows his material. In section after section he takes a set of theorems and, by appropriate concatenation plus some well chosen words of explanation, he creates a Theory.

JOEL SPENCER

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Multidimensional diffusion processes, by D. W. Stroock and S. R. S. Varadhan, Die Grundlehren der mathematischen Wissenschaften, vol. 233, Springer-Verlag, Berlin and New York, 1979, xii + 338 pp., $34.80.

1. *Is it best to think of a 'diffusion' as meaning* (i) *a continuous strong Markov process,* (ii) *a strong solution of an Itô stochastic differential equation, or* (iii) *a solution of a martingale problem*? Both the Markov-process approach and the Itô approach (which holds so special a place in the hearts of probabilists after the appearance of McKean's wonderful book [7]) have been immensely successful in diffusion theory. The Stroock-Varadhan book, developed from the historic 1969 papers by its authors, presents the martingale-problem approach as a more powerful–and, in certain regards, more intrinsic–means of studying the foundations of the subject.

The martingale-problem method has been applied with great success to other problems in Markov-process theory, both 'pure' (Stroock [**10**], . . .) and applied (Holley and Stroock [**3**], [**4**], . . .). It has conditioned our whole way of thinking about still-more-general processes (Jacod [**5**], . . .). Moreover, the method's ideas and results now feature largely in work on filtering and control (Davis [**1**], . . .).

I 'batter' you with the preceding paragraph because the authors make the uncompromising decision not 'to proselytize by intimidating the reader with myriad examples demonstrating the full scope of the techniques', but rather to persuade the reader 'with a careful treatment of just one problem to which they apply'. Halmos's doctrine 'More is less, and less is more' is thereby thoroughly tested; but if one had to choose a single totally-integrated piece of work which in depth and importance shows that probability theory has 'come of age', it would surely be the theorem towards which so much of this book is directed–or perhaps Stroock's extension of it [**10**]. Most of the main tools of stochastic-process theory are used, after first having been honed to a sharper edge than usual. But it is the formidable combination of probability theory with analysis (in the form of deep estimates from the theory of singular integrals) which is the core of the work.

The book's importance has persuaded me to write a review accessible to the general reader–I even define *continuous martingale*! I therefore concentrate on background material, with just a few clues (some for the sharp-eyed!) on how the book's results are proved.

2. The following purely-analytic (and very special) corollary of the main Stroock-Varadhan theorem can help set the scene; and the fact that, in spite of so much research, no analytic proof of its uniqueness assertion has been discovered, can serve as a first indication of the depth of the theorem.

Let L be a second-order elliptic operator on $C_K^\infty(\mathbf{R}^d)$ of the form

$$L = \frac{1}{2}\sum_{i,j<d}\sum a_{ij}(x)\partial_i\partial_j + \sum_{i<d} b_i(x)\partial_i, \tag{2.1}$$

where ∂_i denotes $\partial/\partial x_i$, $\{a_{ij}(x)\}$ is a strictly positive-definite real symmetric matrix for each x, and where each $a_{ij}(\cdot)$ and each $b_i(\cdot)$ is a bounded continuous real-valued function on $\mathbf{R}^d$. No Lipschitz conditions are assumed. Then there is one and only one Feller-Dynkin semigroup $\{P_t\colon t \geqslant 0\}$[1] with infinitesimal generator extending L. (Chapter 10 of the book examines closely the extent to which the 'boundedness' condition on a and b may be relaxed. It cannot be relaxed completely because one must preclude 'explosion'.)

3. With a general Feller-Dynkin semigroup $\{P_t\}$ is associated an $\mathbf{R}^d$ valued Markov process $X = \{X_t\colon t \geqslant 0\}$ with *right-continuous* paths, such that the law P^x of X started from x is given by the usual recipe: for $0 \leqslant t_1 \leqslant t_2 \leqslant \cdots \leqslant t_n$, we have[2]

$$\begin{aligned} &P^x[X_{t_1} \in dx_1; X_{t_2} \in dx_2; \ldots; X_{t_n} \in dx_n] \\ &= P_{t_1}(x, dx_1)P_{t_2-t_1}(x_1, dx_2) \ldots P_{t_n-t_{n-1}}(x_{n-1}, dx_n). \end{aligned} \tag{3.1}$$

But if $\{P_t\}$ has generator extending our operator L, then, because L is *local*, we can take X to be (path-)*continuous*.[3]

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[1]A 'Feller-Dynkin' semigroup $\{P_t\colon t > 0\}$ is a family of bounded linear maps $P_t\colon C_0(\mathbf{R}^d) \to C_0(\mathbf{R}^d)$, where $C_0(\mathbf{R}^d)$ is the Banach space of continuous functions on $\mathbf{R}^d$ which 'vanish at ∞', such that

$$f > 0 \Rightarrow P_t f > 0; \quad P_t 1 = 1; \quad P_s P_t = P_{s+t}; \quad P_\varepsilon f \to f \quad (\varepsilon \downarrow 0).$$

('Pointwise' and 'strong-topology' convergence in the last statement are equivalent under the remaining hypotheses.) The statement that $\{P_t\}$ has *infinitesimal generator extending L* amounts to the statement that

$$P_t f - f = \int_0^t P_s L f\, ds, \quad \forall f \in C_K^\infty(\mathbf{R}^d). \tag{2.2}$$

Note that the differentiated form $(P_t f)' = P_t L f$ of (2.2) corresponds to the Fokker-Planck equation.

[2]Here, the transition probability measure $P_t(x, \cdot)$ is of course that associated with the map $f \mapsto P_t f(x)$ on $C_0(\mathbf{R}^d)$ by the Riesz representation theorem.

[3]Conversely, if each P^x measure associated with a Feller-Dynkin semigroup $\{P_t\}$ assigns mass 1 to the set of continuous paths, then the generator G of $\{P_t\}$ must be *local* and must obey the *maximum principle*: if $f \in D(G)$, then $Gf < 0$ at a maximum of f. Hence if $C_K(\mathbf{R}^d) \subset D(G)$, then G acts on $C_K^\infty(\mathbf{R}^d)$ as a (possibly singular) *second-order* elliptic operator with continuous coefficients. The fact that it can happen that $D(G) \cap C_K^\infty(\mathbf{R}^d) = \{0\}$ is one reason why the definition of a diffusion as a continuous strong Markov process, though intrinsic, is too wide.

A Markov process is a much richer structure than its transition semigroup, and it may be subjected to all sorts of transformations which have no analytic counterpart. This explains why probability theory can obtain analytic results which escape analytic proof (and the literature contains many very striking instances of this). But in the richer setting, the purely-analytic questions are of secondary interest.

4. Continuous P-martingales. We discuss martingale theory only in the special context appropriate to diffusion theory.

Let $\Omega = C([0, \infty), \mathbf{R}^d)$ be the space of all continuous functions ω from $[0, \infty)$ to $\mathbf{R}^d$. For $t \in [0, \infty)$ and $\omega \in \Omega$, write X for the coordinate process with $X_t(\omega) = \omega(t)$. Put $\mathcal{F}_\infty^\circ = \sigma\{X_s: s \geqslant 0\}$, the smallest σ-algebra of subsets of Ω such that every map X_s from Ω to $\mathbf{R}^d$ is $\mathcal{F}_\infty^\circ$ measurable; and, for $t \geqslant 0$, put $\mathcal{F}_t^\circ = \sigma\{X_s: s \leqslant t\}$.

Let M be a continuous adapted real-valued process. Thus, the map M: $[0, \infty) \times \Omega \to \mathbf{R}$ is such that $t \mapsto M_t(\omega)$ is continuous for every ω, and M is $\{\mathcal{F}_t^\circ\}$ *adapted* in that for each t, $M_t(\cdot)$ is $\mathcal{F}_t^\circ$ measurable.[4]

Let P be a probability measure on $(\Omega, \mathcal{F}_\infty^\circ)$. Then M is called a (continuous) *P-martingale* if $M_t \in \mathcal{L}^1(\Omega, \mathcal{F}_t^\circ, P)$, $\forall t$, and, whenever $s \leqslant t$ and $\Lambda \in \mathcal{F}_s^\circ$,

$$\int_\Lambda M_t(\omega) P(d\omega) = \int_\Lambda M_s(\omega) P(d\omega).^5$$

5. Martingale problems for L. Let L be as at (2.1). Let P be a probability measure on $(\Omega, \mathcal{F}_\infty^\circ)$. Let $x \in \mathbf{R}^d$.

DEFINITION. We say that P *solves the martingale problem for L starting from x* if $P[X_0 = x] = 1$ and, $\forall f \in C_K^\infty(\mathbf{R}^d)$, the process C^f, where

$$C_t^f = f(X_t) - \int_0^t Lf(X_s)\, ds$$

is a P-martingale.

Assume for the moment that a Feller-Dynkin semigroup $\{P_t\}$ exists with infinitesimal generator extending L. As explained in §3, it is possible to define a probability measure P^x on $(\Omega, \mathcal{F}_\infty^\circ)$ via (3.1). It is almost trivial to use (2.2) to show that P^x solves the martingale problem for L starting from x.

In the special context corresponding to §2, the main Stroock-Varadhan theorem takes the following form.

THEOREM. *Let L be a second-order elliptic operator on $C_K^\infty(\mathbf{R}^d)$ satisfying the hypotheses described at* (2.1). *Then, for each x in $\mathbf{R}^d$, there is one and only one solution of the martingale problem for L starting from x. Moreover, this solution is of the Markovian form P^x derived as at* (3.1) *from the unique Feller-Dynkin semigroup $\{P_t\}$ with generator extending L.*

In fact, for the existence-and-uniqueness part of the result, Stroock and Varadhan allow the matrices $a(\cdot)$ and $b(\cdot)$ to be *time-dependent*, and impose

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[4]The intuitive thrust of this requirement is that $M_t(\omega)$ is determined when the values $X_s(\omega)$ for $s \in [0, t]$ are known.

[5]This is the full statement of the requirement that the conditional P-expectation of M_t, given the information $\mathcal{F}_s^\circ$ about what has happened up to time s, is M_s: under P, the 'game' M is fair.

only a *measurability* requirement on b. This generalisation is not merely an academic matter: it is just what engineers need (see Davis [1], . . .).

Before thinking briefly about how the theorem is proved, let us spend some time in getting an intuitive feeling for continuous martingales and semimartingales.[6]

6. Continuous P-semimartingales. Again let P be a probability measure on $(\Omega, \mathcal{F}_\infty^\circ)$. The integrability condition $(M_t \in \mathcal{L}^1)$ in the definition of 'martingale' is a nuisance. Call a continuous adapted process M a (continuous) *local P-martingale* if, for every n, M^n is a P-martingale, where

$$M_t^n = M_{t\wedge\tau(n)} - M_0, \quad \text{and} \quad \tau(n) = \inf\{s: |M_s - M_0| \geq n\}.$$

(This is one of many instances where I use simple definitions and/or results which work only for continuous processes. Meyer [8] has the 'correct'–that is, *generalisable*–versions).

We now arrive at one of the central concepts of Strasbourg theory. Call a continuous adapted process Z a (continuous) *P-semimartingale* if Z may be written as follows:

$$Z_t = Z_0 + M_t + V_t, \tag{6.1}$$

where M is a continuous local P-martingale with $M_0 = 0$, and V is a (continuous adapted) process with paths of finite variation and with $V_0 = 0$. It is surprising that *the decomposition* (6.1) *is P-unique*: if $Z = Z_0 + M_t^* + V_t^*$, then $P[M_t = M_t^*, \forall t] = 1$.[7]

7. Generalised Itô formula: continuous case. Let Z be a continuous P-semimartingale with decomposition as at (6.1). You can see that V represents the 'drift' of Z away from the fair (local martingale) situation.

By the celebrated *Meyer decomposition theorem*, there is a P-unique continuous adapted process $\langle M, M\rangle$ with nondecreasing paths such that $\langle M, M\rangle_0 = 0$ and $M^2 - \langle M, M\rangle$ is a local P-martingale. Moreover, by a theorem of Kunita, Watanabe, and Doléans, there exists a sequence (n_k) along which, with P-probability 1,

$$\langle M, M\rangle_t = \lim \sum_i \left[M_{t\wedge i2^{-n}} - M_{t\wedge(i-1)2^{-n}}\right]^2 \quad \forall t,$$

[6]The theory of martingales, and of the still-more-important semimartingales, has been developed to a very advanced level by the French school of probabilists led by Meyer. (But we must not forget massive Japanese contributions.) I myself have on occasion been mischievous in comments on 'Strasbourg theory', and I think that Stroock and Varadhan are perhaps just a little mischievous in this regard. Strasbourg publications are generally inclined to be somewhat abstract in spirit. However, they do contain a profound analysis of intuitive thinking; and, with sufficient poetic licence, they may be regarded as in the direct tradition of Lévy's work. Meyer [8] is the definitive work on the Strasbourg theory of stochastic integrals. Dellacherie, Doléans, Letta, and Meyer [2] is a most helpful 'Strasbourg' look at part of the Stroock-Varadhan theory. These two papers are my guides for the next few sections. Incidentally, I am using the long-thought-about Strasbourg notation throughout the review.

[7]An immediate consequence of the P-uniqueness just asserted is that a *continuous* local P-martingale M cannot have paths of finite variation unless it is constant. (*You* supply the 'almost surely' qualification.) This explains why we need a *stochastic integral*. It is important that the mildest regularity requirements on a 'stochastic integral' force us to consider only stochastic integrals relative to (possibly discontinuous) semimartingales.

the limit existing uniformly on compact t-intervals. Infinitesimally,

$$(dZ_t)^2 = (dM_t)^2 = d\langle M, M\rangle_t$$

(in some sense!), so Taylor's theorem leads us to anticipate the generalised Itô formula for the continuous case:

$$df(Z) = f'(Z)\,dZ + \tfrac{1}{2}f''(Z)d\langle M, M\rangle. \tag{7.1}$$

The moral is that a C^2 function f of a continuous P-semimartingale Z is again a continuous P-semimartingale with decomposition

$$f(Z_t) = f(Z_0) + \int_0^t f'(Z)\,dM + \left\{\int_0^t f'(Z_s)\,dV_s + \frac{1}{2}\int_0^t f''(Z_s)\,d\langle M, M\rangle_s\right\}, \tag{7.2}$$

the first of the integrals, a stochastic integral, yielding a local P-martingale.[8] The final two integrals in (7.2) are Stieltjes integrals over s for each ω.

8. Diffusions as semimartingales. If Z is our continuous P-semimartingale as at (6.1), and f is a C_K^∞ function on $\mathbf{R}$, then, by (7.2),

$$f(Z_t) - \left\{\int_0^t f'(Z_s)\,dV_s + \frac{1}{2}\int_0^t f''(Z_s)\,d\langle M, M\rangle_s\right\}$$

is a local P-martingale. (More general martingale problems loom into view; but back to diffusions . . . .)

Take $d = 1$, and $L = \frac{1}{2}a(x)d^2/dx^2 + b(x)d/dx$, where $a(\cdot)$ and $b(\cdot)$ are continuous (but 'measurable' will do!) and $a(\cdot) > 0$. We see that *a measure P on $(\Omega, \mathcal{F}_\infty^\circ)$ solves the martingale problem for L starting from x if and only if $P[X_0 = x] = 1$ and the coordinate process X is a P-semimartingale with decomposition $X = X_0 + M + V$ satisfying*

$$V_t = \int_0^t b(X_s)\,ds, \qquad \langle M, M\rangle_t = \int_0^t a(X_s)\,ds.$$

'Vector' generalisations to $d > 1$ are obvious. See footnote 8!

9. The martingale problem for Wiener measure. Continue with $d = 1$. Take $b \equiv 0$, $a \equiv 1$, $x = 0$. If P satisfies the corresponding martingale problem, then $P[X_0 = 0] = 1$, and X is a local P-martingale with $\langle X, X\rangle_t = t$. A trivial modification of (7.1) shows that $dY_t^\theta = i\theta Y_t^\theta\,dX$, where $Y_t^\theta = \exp(i\theta X_t + \frac{1}{2}\theta^2 t)$. Hence Y is a local P-martingale, and indeed a *P-martingale* because Y_t^θ is bounded by $\exp(\frac{1}{2}\theta^2 t)$. It is immediate to anyone who recognizes the characteristic function of the normal distribution that X *is an* $(\{\mathcal{F}_t^\circ\}, P)$ *Brownian motion*: X_t *is* $\mathcal{F}_t^\circ$ *measurable* $(\forall t)$, *and, under* P, $X_{t+h} - X_t$ *is independent of* $\mathcal{F}_t^\circ$ *and has the normal distribution of mean* 0 *and variance* h.

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[8]With U standing for $f'(Z)$, the stochastic integral $\int U\,dM$ is a limit of Riemann sums $\Sigma U_s(M_{s+h} - M_s)$, *with each M-increment pointing into the future of the U-value.* This is why $\int U\,dM$ is a local martingale. The super-elegant modern (Kunita, Motoo, Watanabe, . . . , Meyer) theory *defines* $\int U\,dM$ as that P-unique continuous local P-martingale starting at 0 such that for every continuous local P-martingale N with $N_0 = 0$, $\langle \int U\,dM, N\rangle_t = \int_0^t U_s\,d\langle M, N\rangle$ up to P-uniqueness, where $\langle M, N\rangle = \frac{1}{2}(\langle M + N, M + N\rangle - \langle M, N\rangle - \langle N, N\rangle)$. See Meyer [8].

Thus P is forced to be Wiener measure. This result, discovered by Lévy (who else?!), is what gave hope for the martingale problem method. Splendid as the Kunita-Watanabe proof just given is, it is tied to this 'freak' case, and so is superseded by the proof in §12 below.

10. Martingale problems and stochastic differential equations. Again take $d = 1$, and assume that P solves the martingale problem starting from x for $L = \frac{1}{2}a(x)d^2/dx^2 + b(x)d/dx$, where a and b are continuous (or just measurable) and $a(\cdot) > 0$.

Put $\sigma(x) = a(x)^{1/2}$, and write

$$dB = \sigma(X)^{-1}\,dM = \sigma(X)^{-1}[dX - b(X)\,dt], \qquad B_0 = 0. \tag{10.1}$$

Then B is a local P-martingale and $(dB)^2 = a(X)^{-1}d\langle M, M\rangle = dt$. By Lévy's theorem, *$B$ is an $(\{\mathcal{F}_t^\circ\}, P)$ Brownian motion. Moreover, the formally-obvious consequence*:

$$dX = \sigma(X)\,dB + b(X)\,dt \tag{10.2}$$

of (10.1) *is easily proved*. (It is obvious from the modern theory in footnote 8.) Note that *we constructed B from X*.

This is the reverse of the situation in Itô theory, where the aim is to study (10.2) *as an equation for an 'unknown' X for a given Brownian motion B*. Under Lipschitz conditions on a and b, Itô solved (10.2) via successive approximation, Picard-style, obtaining X as a B-adapted process (X_t is $\sigma\{B_s: s \leqslant t\}$ measurable). It is not known whether a B-adapted solution X of (10.2) exists when a mere continuity hypothesis is imposed on a and b: *X may require more randomness than B can provide*. To understand what might be going on here, study the still-rather-mystifying *Tsirel'son counterexample* in Liptser and Shiryayev [**6**].

Many aspects of the relationship between Itô theory and S-V theory are subtle and difficult. Chapter 8 of the S-V book is a careful study, following on from important work of Watanabe and Yamada. One of the main results is that 'Itô uniqueness' (watch the formulation!) implies uniqueness in the martingale-problem sense.

11. Why has the martingale-problem method succeeded where others have failed? In regard to *existence theorems*, the reason lies in the 'solidarity' of probability measures on Polish spaces under 'weak' convergence, and in the fact that *the martingale-problem method is ideally suited for establishing 'weak' compactness of families of measures*. This is because martingale inequalities automatically provide modulus-of-continuity properties for application of Arzéla-Ascoli-Prohorov criteria. (That 'practical' weak-convergence results are handled effectively is evidenced by the book's Chapter 11.) Itô's results are 'dense', and we can take limits!

We now turn to the deeper *uniqueness results*. These are deeper than analysis can *formulate*! But the cunning martingale-problem method *stays sufficiently close to analysis to be able to utilise its estimates*. By contrast, the more probabilistic *Itô theory is too subtle for analysis to be of any help*!

12. Thumbnail sketch of proof of the uniqueness result. Let L be as at (2.1) but with $b \equiv 0$. The *Cameron-Martin-Girsanov change-of-measure theorem* allows us to 'add in (measurable) b' later. Since $a(\cdot)$ is continuous, it is locally almost constant. *Localisation techniques* (*probabilistic*–with no analytic counterpart) make it enough to deal with the case: *$a(\cdot)$ is always close to the identity matrix, so L is 'appoximately'* $\frac{1}{2}\Delta$. You can very easily derive directly from the definition that if P^x (*not* known to be Markovian) solves the martingale problem for L starting from x, then, for $\lambda > 0$, and $f \in C_K^\infty(\mathbf{R}^d)$,

$$R_\lambda^x(\lambda f - Lf) = f(x) \quad \text{where } R_\lambda^x(h) = \int_0^\infty dt\, e^{-\lambda t} \int_\Omega h \circ X_t(\omega) P^x(d\omega). \tag{12.1}$$

The idea of perturbation theory is to try to show that R_λ^x is therefore determined because $R_\lambda^x g = (\lambda - L)^{-1} g(x)$, where we try to define

$$(\lambda - L)^{-1} = V_\lambda (I - K_\lambda)^{-1}, \tag{12.2}$$

where $K_\lambda = \frac{1}{2}\Sigma\Sigma[a_{ij}(\cdot) - \delta_{ij}]\partial_i\partial_j V_\lambda$, and $V_\lambda = (\lambda - \frac{1}{2}\Delta)^{-1}$ is the usual Brownian resolvent. The key fact with which we can work is provided by Littlewood-Paley theory (see Meyer [9] for a fine probabilistic proof), namely: *for $p > 1$, the map $\partial_i\partial_j V_\lambda$ from $C_K^\infty(\mathbf{R}^d)$ to $L^p(\mathbf{R}^d)$ extends* (*uniquely*) *to a bounded linear operator on $L^p(\mathbf{R}^d)$*. By holding $a(\cdot)$ everywhere close to the identity matrix, we can arrange that, for all λ in an interval, $\|K_\lambda\|_p < 1$. While (12.2) is now meaningful in $L^p(\mathbf{R}^d)$ terms, we need to make a much more careful analysis (not done in this review!) to show that *for $p > \frac{1}{2}d$, R_λ^x is a bounded linear functional on $L^p(\mathbf{R}^d)$*. Only then can we say that R_λ^x is determined.[9] By 'inversion of Laplace transforms', *$P^x \circ X_t^{-1}$ is uniquely determined for all t* (*and this for every x*). *On conditioning the martingale-problem formulation relative to $\mathcal{F}_t^\circ$ in the sense of regular conditional probabilities,* we find that

$$[P^x \circ X_{t+h}^{-1} | \mathcal{F}_t^\circ] = P^{X(t)} \circ X_h^{-1} \qquad (P^x \text{ almost surely}).$$

The 'full' uniqueness of P^x and its Markov property (*the two are inextricably linked*) *follow immediately*. (For *Markov selection principles* in the presence of nonuniqueness, see Chapter 12 of the book.)

For their more general and much fuller results, Stroock and Varadhan use much deeper inequalities due to Jones, Fabes, and Rivière. The book's Appendix proves these estimates, and links such results to the theory of *BMO*, etc.

13. I have emphasised the great importance of the Stroock-Varadhan book. It contains a lot more than I have indicated; in particular, its many exercises contain much interesting material.

For immediate confirmation of the subject's sparkle, virtuosity, and depth, see Mozart–sorry, I mean *McKean*–[7]. The Stroock-Varadhan book proceeds on its inexorable way like a massive Bach fugue. 'Too much counterpoint; and, what is worse, *Protestant* counterpoint', said Beecham of Bach. But old

[9] Argue (via localisation) that we can assume that $a(\cdot)$ is constant far out. Hence we can approximate $a(\cdot)$ uniformly by smooth $a_k(\cdot)$. Now it is fairly easy to show that if $g \in C_K^\infty$, then we can find f_n in C_K^∞ with $\lambda f_n - Lf_n \to g$ in L^p. Thus, since R_λ^x is continuous on L^p, $R_\lambda^x g$ may be determined from (12.1).

J. S. can be something of a knockout if his themes get hold of you. And his influence on what followed was (you may say) substantial.

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Computational analysis with the HP 25 pocket calculator, by Peter Henrici, John Wiley, New York, 1977, 280 pp.

Compact numerical methods for computers: linear algebra and function minimization, by J. C. Nash, John Wiley & Sons, New York, 1979, x + 227 pp., $27.50.

LINPACK: User's guide, by J. J. Dongarra, J. R. Bunch, C. B. Moler, and G. W. Stewart, Society for Industrial and Applied Mathematics, Philadelphia, 1979 368 pp., $14.00 list, $11.20 SIAM members.

For sometime beginning in the 1930's, Mathematics and Electrical Engineering had a fruitful liaison. From this liaison a new discipline, sometimes called Information Processing but more often Computer Science, was born. The infant discipline grew rapidly in strength and knowledge, and before long decided to set up its own establishment. Computer Science continued to prosper in this office-home which is today a model of industry and affluence. In it may be found many tools adapted from instruments invented by its parent disciplines, the ardor of whose liaison has meanwhile cooled.

The mathematical tools of Computer Science include a new concept of *real arithmetic*. The set of 'real numbers' in a computer is finite, and disjoint from