4. Epilogue. "We view this book"—write Dreben and Goldfarb—

> "as a prologomenon to an abstract study of solvability and related notions, a study not concerned with particular classes. Is there an informative general criterion that distinguishes those syntactic restrictions that do from those syntactic restrictions that do not lead to solvable classes? We hope our examination of the structural properties of expansions provides the data and tools needed to attack this question, and points to the general concepts in terms of which an answer might be formulated."

It seems pretty clear to me that the desired informative general criterion does not exist, and that this negative answer can be proved when the question is formalized in a reasonable way. An old theorem of Tarski says for example that $\{S$: the set of logically valid implications $S \to S'$ is decidable$\}$ is undecidable.

I do not know how bright the future of the classical decision problem is. It seems clear to me however that the wealth accumulated during years of research should be properly exposed. A comprehensive account of the decision problem for prefix-similarity classes still needs to be written. It should treat satisfiability and finite satisfiability, cases with or without equality, cases with or without function symbols. Other most important programs should be comprehensibly presented. Complexity of the decision procedures should be treated, and connections between the classical decision problem and other subjects (computer science in particular) should be shown.

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BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 7, Number 1, July 1982

0273-0979/82/0000-0456/$01.50

Pseudodifferential operators, by Michael E. Taylor, Princeton Univ. Press, Princeton, N. J., 1981, 451 pp., $35.00.

The theory of partial differential equations, even only the linear ones, is one of the vastest in mathematics. It has a great variety of subjects to study and an even greater variety of tools, ranging from very general principles to really special tricks. Most authors of books on this subject therefore deal with only a rather limited number of topics. If well chosen these can be representative for a reasonable part of the theory as a whole. In particular if it is the intention to write a textbook then this is a wise strategy. To aim at greater completeness requires not only a broad and deep knowledge of the field but also a heroic stamina.

The book of Taylor belongs to the unusual class where completeness has been a major goal. It treats almost all the important tools of linear partial differential equations, from Sobolev spaces modeled on L^2 and L^p to Hölder and Besov spaces (using interpolation as one of the guiding principles). From Fourier analysis to pseudodifferential, Fourier-integral, and Fourier-Airy-integral operators. From a priori inequalities to propagation of singularities to Tauberian theorems. Also the variety of problem to which the methods are applied is impressive and amply representative for the richness of the field. Initial value problems for hyperbolic equations are treated by Friedrichs' symmetrizer technique and an elegant reduction to first order systems using pseudodifferential operators. This reduction is also used to treat elliptic boundary value problems. Here there is an emphasis on obtaining the a priori estimates whereas I would have preferred a presentation of the Caldèron projection. The reflection of singularities at the boundary gets a nice treatment with the author's decoupling trick. I also like his bold use of all kinds of analytic functions of a positive selfadjoint operator, as for instance in his joint work with J. Cheeger on diffraction by conical singularities. In the chapter on grazing rays we see one of the specialists at work. It would have been nice though if the geometric approach of R. Melrose, the other specialist, would have been used to further enlighten the reader.

In general the style of the book is computational rather than geometric, reflecting the author's special talents of course. However, in the last decade geometry has proved to be a very fruitful guide in this field, and I find this somewhat underrepresented in this otherwise so rich book. The final chapter, on operators with double characteristics, gives a fine presentation of the state of the art. Also some nonspecialistic excursions are made, like the paragraphs on harmonic analysis on compact Lie groups. Here pseudo-differential operators are used to obtain the asymptotic behavior of the multiplicities of K-irreducible representations in the restrictions to K of an irreducible representation of G. Here K is a compact subgroup of a compact Lie group G. For a Lie group speciàlist the challenge is of course to derive these asymptotics directly from the explicit Weyl character formula, and in fact this has been done by Heckman in his 1980 Leiden thesis. On the other hand it seems to be the general experience that all the explicit formulae in Lie group theory do not provide much better asymptotic estimates than the general methods.

The book is an extreme example of a research monograph as opposed to a textbook. In the first chapters the reader is confronted with an extremely rapid succession of definitions and basic results, with almost no time spent on philosophizing the subtleties or historical background. For instance, in a book on pseudodifferential operators one expects at least a paragraph explaining that one is dealing with integral operators with a kernel having singularities along the diagonal $x = y$. Also one would like to see an analysis of the nature of these singularities, and the philosophy that it is easier to work with the Fourier decomposition of these singularities rather than with the kernels themselves. Not even the classical potentials get any attention!

Sometimes one gets the impression that function spaces are introduced just for the technical fun of it. As an example, Hölder spaces appear but not the nonlinear elliptic equations for which early in this century they were discovered to be the best tool. The only nonlinear equations treated in the book are quasilinear hyperbolic systems. There L^2-type estimates have to be used, making the assumptions less natural and general than for nonlinear elliptic equations in the Hölder space setting. A confrontation of these two cases would have been very enlightening for the student.

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BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 7, Number 1, July 1982

0273-0979/82/0000-0162/$01.25

K-theory of forms, by Anthony Bak, Annals of Mathematics Studies, vol. no. 98, Princeton Univ. Press, Princeton, N. J., 1981, viii + 268, Cloth $20.00, paperback $8.50.

The study of quadratic forms over general rings is a recent phenomenon. Until the mid 1960s only quadratic forms over rings of arithmetic type were considered, and the subject was a branch of number theory. All this changed with the development of algebraic K-theory and its application to the topology of manifolds at the hands of the surgery obstruction theory of Wall [5]. The surgery obstruction groups $L_*(\pi) = L_*(\mathbf{Z}[\pi])$ consist of stable isomorphism classes of quadratic forms over the integral group ring $\mathbf{Z}[\pi]$ of a group π, and also of the stable unitary groups of automorphisms of such forms. For finite π the computation of $L_*(\pi)$ is just about possible, and Bak has been one of the leading researchers in the field. The computations of all authors are based on the localization-completion "arithmetic square" of a finite group ring $\mathbf{Z}[\pi]$

$$\begin{array}{ccc} \mathbf{Z}[\pi] & \to & \mathbf{Q}[\pi] \\ \downarrow & & \downarrow \\ \hat{\mathbf{Z}}[\pi] & \to & \hat{\mathbf{Q}}[\pi] \end{array}$$

in which the other rings are quite close to being of arithmetic type, and for which there is an algebraic Mayer-Vietoris sequence in the L-groups of the type

$$\cdots \to L_n(\mathbf{Z}[\pi]) \to L_n(\mathbf{Q}[\pi]) \oplus L_n(\hat{\mathbf{Z}}[\pi]) \to L_n(\hat{\mathbf{Q}}[\pi]) \to L_{n-1}(\mathbf{Z}[\pi]) \to \cdots$$

generalizing the classical Hasse-Minkowski local to global principle in the unstable classification of quadratic forms over global fields.

The book under consideration is a collection of all the definitions and general theorems in the algebraic K-theory of forms which are required for the author's computations, and as such it is very welcome. Unfortunately, I doubt if the reader who is not interested in the background of Bak's computations will get much out of this book. So many different types of unitary K-groups are defined (the index lists 40) that it is practically impossible to keep track of them, especially as there are no examples given of any kind to show that they are distinct and nonzero. The absence of feeling for the history of the subject also makes the book hard to read. For example, it would help if the motivation in §1.D for the introduction of "form parameters" not only mentioned the