that goes beyond ZFC, and this often confuses sharp-witted beginners. One way out is as follows: introduce M as a formal constant into the language of set theory and let T be the theory in the larger language whose axioms are those of ZFC plus all sentences obtained by relativizing each of the axioms of ZFC to the set M. It follows from the Reflection Principle (which is in turn an important consequence of the Replacement Axiom) that T is a conservative extension of ZFC. That is, any sentence which is provable in T and which does not mention M can already be proved in ZFC; in particular, T is consistent. It is within T that one constructs the forcing extension N of M. In proving that a given assertion φ is true of N, one must identify a finite set of ZFC axioms which one assumes to hold in M and in the universe of sets as well. Then φ is proved within the corresponding finitely axiomatized subtheory of T.

In the course which I taught from this book and have outlined here, my students found it necessary to hop around in the book quite a bit. Although I worried about this, they did not, and their consensus was that the book is demanding but readable. They especially liked the extensive indexing and cross-referencing which the author has provided.

We found no serious mistakes and only a few misprints, most of them easily detected and corrected. One which deserves special mention is that the ordering relation $q \leqslant p$ in part (b) of Definition 2.4 (p. 53), which is fundamental to the forcing construction, should instead be $p \leqslant q$.

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The representation theory of finite groups, by W. Feit, North-Holland Publishing Co., Amsterdam and New York, 1982, xiv + 502 pp., $55.00 (Dfl. 140.00). ISBN 0-4448-6155-6

The general modular theory of representations of finite groups is the subject of Walter Feit's book, *The representation theory of finite groups*. The theory began with the work of Richard Brauer in the 1940s. Its goals were expressed in Brauer's paper of 1944, *On the arithmetic in a group ring*:

> "We are far from knowing all important properties of group characters. In particular, we are interested in further results which connect the group characters directly with properties of the abstract group G. Any result of this kind means, in the last analysis, a result concerning the structure of the general group of finite order. One approach to our question is to study arithmetic properties...."

That motivation still holds today, joined of course by a number of other ones as the theory developed over the years.

In the classical theory of representations of a finite group G, characters correspond to finite-dimensional $K[G]$-modules V, where K is an algebraically closed field of characteristic 0 and $K[G]$ is the group algebra of G over K. In practice it may be assumed that K is a sufficiently large algebraic number field. To study the deeper properties of characters, Brauer decomposed them modulo a prime. For that and other purposes, K may be assumed to be complete with respect to a prime ideal divisor $\mathfrak{p}$ of the rational prime p. If R is the ring of $\mathfrak{p}$-local integers in K, an $R[G]$-lattice L, that is, an R-free, $R[G]$-module, can be chosen in the $K[G]$-module V so that the characters χ_V and χ_L of V and L coincide, in the sense that they agree on G. The quotient module $\bar{L} = L/\mathfrak{p}L$ then has the structure of an $\bar{R}[G]$-module, where $\bar{R} = R/\mathfrak{p}$ is a finite field of characteristic p. Its decomposition gives the modular decomposition of χ_V. Three group rings, $K[G]$, $R[G]$, $\bar{R}[G]$, then occur in this theory with $R[G]$ playing a pivotal role. Since the theory has significance only when p divides the order of G, a peculiar situation arises. Reduction modulo $\mathfrak{p}$ in this theory, unlike its use in number theory, studies a simpler structure by more complicated ones, namely $K[G]$ and representations over K by $R[G]$ and $\bar{R}[G]$ and representations over R and $\bar{R}$.

The theory is also called block theory because of the fundamental part played by blocks. Blocks arise from the decomposition of the group algebra

$$\bar{R}[G] = \bigoplus_B B \tag{1}$$

into a direct sum of indecomposable 2-sided ideals, the B being called the block ideals. Since R is complete, idempotents of $\bar{R}[G]$ lift to idempotents of $R[G]$. In particular, the primitive idempotent decomposition $\bar{1} = \Sigma_B \bar{e}_B$ of the identity of $\bar{R}[G]$ corresponding to (1) lifts to a corresponding decomposition $1 = \Sigma_B e_B$ of the identity of $R[G]$. The indecomposable $R[G]$-modules V are then put into blocks as follows: With abuse of notation, we put V into the block B if $V = Ve_B$. This applies, in particular, to indecomposable $R[G]$-lattices and to indecomposable $\bar{R}[G]$-modules, since the latter can be viewed as $R[G]$-modules. It also applies to the irreducible characters of G. For if χ_V is the character of an irreducible $K[G]$-module V, then $\chi_V = \chi_L$ for some indecomposable $R[G]$-lattice L in V. We then put χ_V into the block B containing L. Of course, B then depends only on χ_V and not on the choice of L.

A group-theoretic invariant connected with B is the defect group of B, a p-subgroup D of G determined up to conjugacy in G. The relation of D to the modules in B remains one of the main problems in the theory. A number of conjectures posed by Brauer continue to fascinate the experts.

Among the many remarkable discoveries made by Brauer, two were singled out by him as the First and Second Main Theorems. The First Main Theorem establishes a 1-1 correspondence between blocks of G with defect group D and the blocks of the normalizer $N(D)$ with defect group D, the correspondence being induced by a Brauer homomorphism $\mathrm{Br}^G_{N(D)}$. The homomorphism Br^G_H is an algebra homomorphism from the center $Z(\bar{R}[G])$ of $\bar{R}[G]$ into the center $Z(\bar{R}[H])$ of $\bar{R}[H]$, defined for suitable subgroups H of G. In the First Main Theorem, blocks B and b of G and $N(D)$ correspond if and only if the idempotents $\bar{e}_B$ and $\bar{e}_b$ are related by the equation $\mathrm{Br}^G_{N(D)}(\bar{e}_B) = \bar{e}_b$.

The Second Main Theorem concerns the values of characters on elements of the form xy, where x is a fixed p-element of G and y is any p'-element in the centralizer $C(x)$. These terms mean that x has order a power of p and y has order relatively prime to p. In order to state the theorem, we need the notion of the Brauer character of an $\bar{R}[G]$-module V. The values of the trace function tr_V of V on p'-elements of G are sums of p'th roots of unity. Since the residue class map $R \to \bar{R}$ induces a 1-1 correspondence between the p'th roots of unity in R and those in $\bar{R}$, these values can be lifted to R. The R-valued function φ_V, defined on the p'-elements of G obtained in this way, is called the Brauer character of V. We put φ_V into the block of G containing V. Given an irreducible character χ of G, there exist algebraic integers $d_{\chi\varphi}$ in R, depending only on x, χ, and φ such that

$$\chi(xy) = \sum_{\varphi} d_{\chi\varphi}\varphi(y) \tag{2}$$

for all p'-elements y in $C(x)$. The φ in (2) run over the Brauer characters of irreducible $\bar{R}[C(x)]$-modules. The Second Main Theorem states that $d_{\chi\varphi} \neq 0$ only if the block B of G containing χ and the block b of $C(x)$ containing φ are related by the equation $\mathrm{Br}^G_{C(x)}(\bar{e}_B)\bar{e}_b = \bar{e}_b$.

Brauer stated most of his discoveries in terms of characters. In more recent developments in the theory, ring and module theories have played increasingly larger roles. One may see this in one of the deepest achievements of the theory, that concerning blocks with cyclic defect groups. In the case of blocks with defect groups of order p, Brauer obtained almost complete information. Among other things, he found a beautiful description of the modular decomposition of the irreducible characters in the block by properties of a tree. The extension by Dade of this work to the general cyclic case came 25 years later, and required new methods of Green and Thompson which work on the level of modules, but not on the level of characters.

In 1959 J. A. Green introduced the notion of a vertex and a source of an indecomposable $R[G]$-lattice or $\bar{R}[G]$-module. The vertex of such a module V is a subgroup P of G minimal with respect to the property that V is $R[P]$- or $\bar{R}[P]$-projective in the sense of relative projective modules in ring theory. P is then a p-subgroup of G determined up to conjugacy in G. The source of V is an indecomposable $R[P]$-lattice or $\bar{R}[P]$-module S such that V is a direct summand of the induced module S^G. The isomorphism class of S is unique up to a conjugate action by an element of $N(P)$. Green also established a 1-1 correspondence between indecomposable G-modules V with vertex P and indecomposable $N(P)$-modules with vertex P. Properties of V are reflected in its Green correspondent, but the exact relationships between V, its vertex and source, and its Green correspondent remain open questions.

The obvious parallels between the Brauer and Green theories do in fact come from a number of relations between them. For example, if a block ideal B of $\bar{R}[G]$ is viewed as an indecomposable $\bar{R}[G \times G]$-module, then the diagonal embedding of a defect group of B into $G \times G$ is a vertex of B, and the Brauer and Green correspondences are naturally related. It is interesting to see the form given by Nagao to the Second Main Theorem in the context of modules:

Let L be an indecomposable $R[G]$-lattice in a block B. Let x be a p-element of G. Then the restriction $L_{C(x)}$ of L to $C(x)$ decomposes as a sum $L_1 \oplus L_2$ of $R[C(x)]$-lattices L_1 and L_2, where the indecomposable components of L_1 belong to blocks b of $C(x)$ satisfying $\mathrm{Br}^G_{C(x)}(\bar{e}_B)\bar{e}_b = \bar{e}_b$, and where the indecomposable components of L_2 have vertices not containing x.

In recent years a rethinking of the basic ideas of the modular theory in terms of the basic structures $\bar{R}[G]$, $R[G]$, $K[G]$ has also taken place. With the insight gained by the earlier points of view, it has been possible to formalize some parts of the theory, thereby clarifying some of its features. For example, Alperin and Broué have constructed a Sylow theory for the set of pairs (P, b), where P is a p-subgroup of G and b is a block of $C(P)$. The First Main Theorem can then be interpreted as a Sylow theorem. The Second Main Theorem, in its newest guise, becomes a statement on the commutativity of two maps.

The theory described so far is general in that it applies to all finite groups. The further elaborations of the theory when the groups are Coxeter or Chevalley groups are not part of the general theory as such. These elaborations exhibit striking compatibility with the Young theory of representations of the symmetric groups and with the Lie theory and the Deligne-Lusztig theory of representations of the Chevalley groups. But that is another story.

In the introduction to his book, which was completed in 1980, Feit writes that his aim is "to give a picture of the general theory of modular representations as it exists at present." His treatment, a personal, yet universal, account, succeeds admirably. Almost every development in the general theory during the then 40 years of its existence is commented upon with at least a bibliographical reference and fitted into a coherent scheme in his presentation. The bibliography of nearly 500 items is remarkably complete. Perhaps the book passes somewhat quickly over the recent formalizations of the theory. On the other hand, it tarries on the Brauer theory, and especially on the beautiful theory of blocks with cyclic defect groups. Any account of the cyclic theory must contend with inherent difficulties in the subject. In this book additional hardships are assumed by not requiring that K or $\bar{R}$ be splitting fields for G. A price is paid for this extra generality, but the development is as complete as the reader could ever hope to have or to use. A number of important applications are given after the exposition of the theory. These include a study of permutation groups of degree p and linear groups of degree at most p, a study in which the cyclic theory plays a prominent part; an analogue in characteristic p due to Brauer and Feit of Jordan's theorem on the existence of normal abelian subgroups of bounded index; and the Glauberman Z^*-theorem on the embedding of involutions in groups.

The writing is spare yet elegant, slanted towards the specialist, yet self-contained. Readers familiar with Feit's *Characters of finite groups* will know the style. Misprints do occur at awkward places in the text, and the lack of punctuation may distract at first. But these are minor matters and will not deter the reader from the best account of the general theory.

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Nonlinear oscillations, dynamical systems, and bifurcations of vector fields, by John Guckenheimer and Philip Holmes, Applied Mathematical Sciences, Vol. 42, Springer-Verlag, New York, 1983, xvi + 453 pp., $32.00. ISBN 0-3879-0819-6

Smooth dynamical systems, by M. C. Irwin, Academic Press Inc., London, 1980, x + 260 pp., $48.50. ISBN 0-1237-4450-4

Geometric theory of dynamical systems, an introduction, by Jacob Palis, Jr. and Welington de Melo, Springer-Verlag, New York, 1982, xii + 198 pp., $ 28.00. ISBN 0-3879-0668-1

In 1963 E. N. Lorenz wrote an article [L] discussing a meteorological model given by the system of differential equations:

$$\frac{dx}{dt} = -10x + 10y, \quad \frac{dy}{dt} = 28x - y - xz, \quad \frac{dz}{d} = -\frac{8}{3}z + xy.$$

(The constants are actually values chosen by Lorenz of certain parameters.)

It is a remarkable fact that fifty years ago it would have been impossible to make more than a superficial analysis of the behavior of the solutions of this system, while today there is an extensive literature concerning it (including numerous articles and at least one book [S]). It is also the case that no standard text on ordinary differential equations contains anything of particular value in understanding this system. In contrast, the three books under review are of considerable value, in the sense that they would go a long way toward preparing the reader to read the literature. And, in fact, the book by Guckenheimer and Holmes discusses precisely this system in considerable detail.