mathematics. The exercises are many and wonderful, leading the reader through dozens of interesting examples, omitted proofs, and explicit calculations.

If any mathematician *can* read it, who should? The authors suggest in the introduction that their book might serve as an introduction to the weightier tomes [5, 6], for a student studying semisimple harmonic analysis. This is certainly a possibility; but Knapp's book especially is already quite accessible, and many of the wonderfully particular things about $SL(2)$ discussed by Howe and Tan are not really necessary or helpful for understanding the general theory. (To be fair, the authors might argue that this reveals flaws in the general theory.)

But the authors also speak of their book as "a day hike to a nearby waterfall", and this is a better guide to who should read it. Every chapter is full of beautiful mathematics that is not as well known as it deserves to be. If you like waterfalls, come and have a look.

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David A. Vogan, Jr.
Massachusetts Institute of Technology
E-mail address: dav@math.mit.edu

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Classical recursion theory, by P. Odifreddi. North-Holland, Amsterdam, 1992, xvii+668 pp., $63.00. ISBN 0-444-89483-7

Recursion theory as we know it today was born in the head of Alonzo Church one day in 1934. Church and his students, in an (eventually unsuccessful) effort to axiomatize the notion of a function, had arrived at the notion of a lambda definable function. A rather trivial consequence of the definition was that every lambda definable function was computable; i.e., the value of the function could be computed on a computer using the arguments as input. (Everyone who has programmed realizes that more powerful computers do not compute more functions; they simply compute the old functions faster and more easily.) Church had the radical thought that all computable functions were lambda definable. This was a remarkable foresight, since at that moment it had not yet been proved that the function $f(x) = x - 1$ was lambda definable. His student

Kleene proved that this and many other computable functions were lambda definable; this, together with some analysis of computable functions by Turing, eventually convinced almost everyone that Church's idea, now known as Church's thesis, was true. Putting aside the details, Church immediately showed that his thesis had remarkable consequences; certain problems of considerable interest could be shown not to have an algorithmic solution (i.e., one which could be programmed on a computer). Until then, no one had any idea of how to prove such a thing about any problem.

Church then left the subject, but Kleene and others continued to develop a general theory of recursive functions (as the lambda definable functions came to be called). In some parts of this theory, such as degrees, computability continued to be central. In others, definability became the central concept. Obviously every computable function is definable—it is the function defined by a certain algorithm—but many other collections of definable functions arise in recursion theory. The study of these gives rise to the Kleene hierarchies.

Of particular interest are classes defined by recursion; i.e., by taking the smallest class of functions (or perhaps sets) that satisfies certain closure properties. The class of recursive functions can be defined in this way; hence the name. In many cases, a class defined by recursion turns out to have other definitions, involving computability, definability, or other notions. Many of the deepest and most interesting results in recursion theory concern this equivalence between different types of definitions.

If a function is to be computable, its arguments and values must be something that can be stored in a computer; this means that they must essentially be integers. The other topics we mentioned, however, make sense for more general arguments or values, e.g., ordinals or sets. This leads to generalized recursion theory, as opposed to the classical recursion theory we have been discussing. Odifreddi's book is an attempt to give fairly complete coverage of the classical theory. Although it is not apparent from the title page, this is the first volume of a proposed three volume work. Even this is not really enough room; the author must frequently sketch proofs or say the proof "is an obvious generalization of the proof of ... ". With these short cuts, he manages to cover a huge number of topics quite thoroughly. (Some of these have not received attention in years and might have been allowed to rest in peace; but many still seem quite interesting.)

What I missed most in this book was a clear delineation of what are the essential lines of recursion theory and what are merely curious questions or ingenious tricks. For this reason, I cannot recommend it as an introduction to the subject, but, as a reference for the recursion theorist who is not too far out in the realm of generalized recursion theory, it could be quite valuable.

J. R. Shoenfield
Duke University
E-mail address: jrs@duke.math.edu