bit wordy and rambling. That, however, is only a minor criticism of a valuable work of scholarship.

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An introduction to harmonic analysis on semisimple Lie groups, by V. S. Varadarajan. Cambridge Studies in Advanced Math., vol. 16, Cambridge Univ. Press, Cambridge and New York, 1989, x+316 pp., $69.50. ISBN 0-521-34156-6

Semisimple Lie groups are symmetry groups that occur in surprisingly many situations. They are the isometry groups of Riemannian symmetric spaces, the analytic automorphism groups of bounded symmetric domains, the groups from which Eisenstein series and cusp forms are constructed in analytic number theory, the conformal groups of general relativity, the groups whose representations correspond to elementary particles, ... They should form part of the basic toolkit of every modern mathematician; but, in fact, the theory is relatively unknown because it is not easily accessible.

The reason for the inaccessibility of semisimple Lie group theory is clear to anyone who has tried to learn or to teach it: One must navigate a path too complicated to follow without a good vehicle and a good map. But it's well worth the effort: that path leads to a breathtaking mathematical vista.

Varadarajan's book *Harmonic analysis on semisimple Lie groups* is the best introduction to harmonic analysis on semisimple Lie groups from the analytic viewpoint. It is neither a textbook nor a monograph in the usual sense, but rather a sort of pedagogic discourse that exposes the reader to semisimple Lie theory in a useful and informative way. After reading this book, one can either stop with a pretty good understanding of the theory and its role in harmonic analysis (if not in geometry, probability, or physics), or one can continue to study the theory with a reasonable background and an excellent sense of direction. Also, and this is no small matter, the book is a pleasure to read.

There are other important viewpoints for harmonic analysis on semisimple Lie groups and their homogeneous spaces. One has a viewpoint oriented toward linear algebraic groups that includes the theory of p-adic semisimple groups, a viewpoint oriented toward number theory that includes automorphic representation theory, a viewpoint of Riemannian geometry and symmetric spaces, a viewpoint of particle physics, and their various mixtures. But life is too short to discuss those here.

Varadarajan's book begins with an interesting introduction to harmonic analysis, with reference to classical Fourier analysis and several accessible applications. Then the book starts in a serious way with a quick sketch of harmonic analysis on compact groups. The Peter-Weyl Theorem, which extends the method of harmonic analysis from Fourier development of periodic functions of one variable to functions on general compact topological groups, is described with emphasis on representations of the group. Weyl's character formula and the Plancherel formula, in terms of Cartan's highest weight theory, are indicated for the unitary group. These methods, and the connection between compact and complex groups, then are briefly indicated in general. This is followed by a concise sketch of some basic facts on unitary representations and harmonic analysis for locally compact groups. Varadarajan gives a brief history, says some words about Haar measures on groups and quotient spaces, sketches the theory of induced representations, says a few words about factor representations, and concludes:

> This is the historical background of our subject. ... its thrust is very clear; *classical Fourier analysis is a very uncertain guide in predicting how noncommutative harmonic analysis should be done.* It is thus not surprising that already in the middle and late 1940s some people were turning their attention to some of the simplest noncompact nonabelian groups like $\mathrm{SL}(2; R)$ and $\mathrm{SL}(2; C)$, to get a better insight into the new phenomena which arise. The work of Gel'fand-Naimark, Bargmann, and Harish-Chandra dates to this period and marks, in my opinion, the real beginning of modern noncommutative Fourier analysis and representation theory.

Every mathematician should know the material in the three chapters just described.

The remaining four chapters are difficult but rewarding. They start with a short sketch of infinitesimal (Lie algebra) methods. Those methods are later seen to lead to the differential equations that make possible Harish-Chandra's finiteness theorems and character theory and the resulting a priori estimates on matrix coefficients. The character theory and estimates are Harish-Chandra's analytic link between harmonic analysis and representation theory. Then there is a thorough analysis of the irreducible modules for the groups $\mathrm{SL}(2; C)$ and $\mathrm{SL}(2; R)$. They are the simplest noncompact nonabelian groups. $\mathrm{SL}(2; R)$ is the multiplicative group of 2×2 real matrices of determinant 1, with the structure of a 3-dimensional real analytic manifold derived from its natural embedding in the R^4 of 2×2 matrices, and $\mathrm{SL}(2; C)$ is its complexification. With this setup established, the last three chapters start with the Plancherel formula for complex semisimple Lie groups (less technical than the general case of real semisimple Lie groups), a description of the discrete series and its contribution to the Plancherel formula for real semisimple Lie groups, and an illustration by specialization to $\mathrm{SL}(2; R)$. Then Varadarajan sets up the differential equations that lead to the a priori estimates mentioned above. Finally, he illustrates those estimates, and their central role in harmonic analysis, with $\mathrm{SL}(2; R)$. This last chapter is not as successful as the earlier chapters because the material is intrinsically more difficult, but still it is the best treatment available.

There are two fundamentally different approaches to analytic aspects of harmonic analysis on semisimple Lie groups. Of course, each has influenced the development of the other. One grew out of physics and developed a lot of specific information about particular representations of certain low-dimensional semisimple Lie groups. This approach is represented, for example, by the early work of Gel'fand, Naimark, and Graev. The other grew out of structure theory and presents a coherent general picture of harmonic analysis on real semisimple Lie groups and some of their homogeneous spaces. The general theory for real semisimple Lie groups was established by one person, namely, Harish-Chandra, who trained as a physicist. It is perhaps best described in Garth Warner's two volume work cited below and in Harish-Chandra's three-paper series on harmonic analysis on real reductive groups. The specific picture for the particular group $SL(2; R)$ is a key component of the general theory. In my view, the general structure-based theory is the pinnacle of this development, and specific information on specific groups is useful, in fact necessary, information. This book by Varadarajan is a good first step toward learning that general theory, indicating the lines of that theory, and illustrating those considerations by developing harmonic analysis on the group $SL(2; R)$.

Since Varadarajan's book describes harmonic analysis on the group $SL(2; R)$, it is worth taking a moment to compare his treatment of $SL(2; R)$ with the treatments in the books of Lang and of Sugiura cited below. Lang goes straight for the Plancherel formula on $SL(2; R)$ and for analysis on the quotient $SL(2; R)/SL(2; Z)$ by the discrete subgroup consisting of integer matrices. His proofs are complete and efficient, he minimizes reference to representation theory, for the sake of efficiency he uses many arguments that only work with $SL(2; R)$, and he does not explicitly indicate how things go for semisimple Lie groups in general. The treatment of $SL(2; R)$ in Sugiura's book also avoids general semisimple representation theory. It is more of a summary than in Lang's book, but it has a somewhat more analytic viewpoint. By contrast, Varadarajan tries to explain the general picture (at least for linear real semisimple Lie groups) and illustrates with the case of $SL(2; R)$. He does not hesitate to leave out details in order to clarify the structure or idea of an argument, and he avoids methods that do not work with real semisimple Lie groups in general.

It is always a problem to set up a course of study for a potential student in semisimple harmonic analysis. Some of the books that one might use are

- S. Helgason, *Differential geometry, Lie groups and symmetric spaces*, Academic Press, New York, 1978.
- S. Helgason, *Groups and geometric analysis*, Academic Press, New York, 1984.
- J. Humphreys, *Introduction to Lie algebras and representation theory*, Springer, New York, 1972.
- A. W. Knapp, *Representation of semisimple groups—An overview based on examples*, Princeton Univ. Press, Princeton, NJ, 1986.
- S. Lang, $SL(2; R)$, Addison-Wesley, Reading, MA, 1975; Springer, New York, 1985.
- M. Sugiura, *Unitary representations and harmonic analysis—an introduction*, second ed., North Holland, Amsterdam, 1990.
- N. R. Wallach, *Real reductive groups*. I, Academic Press, New York, 1988.
- G. Warner, *Harmonic analysis on semisimple Lie groups*. I, II, Springer, New York, 1972.
- V. S. Varadarajan, *An introduction to harmonic analysis on semisimple Lie groups*, Cambridge Univ. Press, Cambridge and New York, 1989.

- V. S. Varadarajan, *Lie groups, Lie algebras and their representations*, Prentice-Hall, Engelwood Cliffs, NJ, 1974; Springer, New York, 1984.
- D. Vogan, *Representations of real reductive groups*, Birkhäuser, Basel, 1981.
- D. Vogan, *Unitary representations of reductive Lie groups*, Princeton Univ. Press, Princeton, NJ, 1987.

My inclination now would be to have a hypothetical student start with this book of Varadarajan, perhaps supplementing the background material with the first three chapters of Sugiura (for an analyst) or the book of Humphreys (for an algebraist), in an independent reading course. Then I would want the student to take a Lie groups course at the level of Varadarajan's text cited above. Finally this student should study material from Helgason, Knapp, Vogan, Wallach, and/or Warner, depending on his level, background, and interests.

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Numerical methods for conservation laws, by Randall J. LeVeque. Birkhäuser-Verlag, Basel, 1992, 214 pp., $24.50. ISBN 3-7643-2723-5

The theory of numerical methods for nonlinear hyperbolic partial differential equations, or conservation laws, has become one of the great successes of numerical analysis. The development of schemes for nonlinear hyperbolic equations requires an understanding of both numerical analysis and the theory of nonlinear hyperbolic equations. By using knowledge of the structure of the solutions of these equations, methods have been developed that compute highly accurate solutions.

The development of the theory of nonlinear hyperbolic partial differential equations in the last fifty years has been stimulated by the growth in applications such as supersonic aerodynamics, thermonuclear explosions, and oil recovery. In each of these applications the differential equations express the conservation of mass, momentum, and other quantities. The increased power of numerical computations has enabled researchers to study ever more complex physical problems governed by conservation laws. A better understanding of the solutions of the differential equations was needed to develop schemes that would compute accurate solutions. Usually the computations of physical phenomena have been beyond the pale of the theory, serving to motivate further work in the theory.

The theory of conservation laws is a rich part of mathematics. One of the best introductions to this theory is the book by Lax [2]. The basic difficulty with nonlinear hyperbolic partial differential equations is that the solutions develop singularities, especially discontinuities, which are usually called shock waves or