multidimensional problems puts severe constraints on the evolution of data and why this is a serious obstacle to finding flows which will be compatible with linear systems for which the IST technique can be implemented.

The broad field of integrable systems is now enormous. No book could possibly cover all of the important aspects. This book carves out a piece of territory, multidimensional integrable equations and aspects of their solutions, especially those corresponding to decaying initial data, employing the IST method. The book can serve a reader well in many ways. The methods are explained, with lots of references given. Hence interested researchers can access the necessary external papers and books for any required material that is not contained in this book. The book would be a useful addition to a library's collection in a field of study which continues to expand rapidly.

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The theory of semirings (*with applications in mathematics and theoretical computer science*), by Jonathan S. Golan. Pitman Monographs & Surveys in Pure and Appl. Math., vol. 54, Longman Scientific & Tech., Essex, 1992, 318 pp., $140.00. ISBN 0269-3666

There are many concepts of universal algebras generalizing that of an associative ring $(R, +, \cdot)$. Some of them—in particular, nearrings and several kinds of semirings—have proved very useful with respect to their various applications. Nearrings arise from rings by canceling either the axioms of left or that of right distributivity and allowing $(R, +)$ to be also a noncommutative group. The second type of those algebras $(S, +, \cdot)$, called semirings (and sometimes hemirings or halfrings), have in common that, as in the case of rings, both distributive laws are demanded; but $(S, +)$ as well as $(S, \cdot)$ are only assumed to be arbitrary semigroups. They differentiate with respect to miscellaneous further ringlike properties which are included or not by different authors. Some of these properties will be discussed later together with the definition of hemirings and semirings used in the book under consideration. We mention in this context also that a common generalization of nearrings and semirings, called seminearrings, has been investigated in several papers (cf. the references given in the chapters about nearrings and nearfields and about semirings and semifields in Volume 1 of the *Handbook of Algebra*, Elsevier Science Publishers).

Semirings, in the general setting as described above or with more restrictive assumptions, arise naturally in such diverse areas of mathematics as combinatorics, functional analysis, topology, graph theory, Euclidean geometry, ring theory including partially ordered rings, optimization theory, automata theory, formal language theory, coding theory, and the mathematical modeling of quantum physics and parallel computing systems. Moreover, semirings provide the most natural common generalization of rings and distributive lattices.

Historically, the semiring of natural numbers, including the zero or not, is surely the oldest algebraic structure in which mankind has calculated; and the construction of rational numbers via the ring of integers or, as in school education, via the semifield of nonnegative rationals, are special cases of more general investigations in semiring theory. In the literature semirings appeared implicitly for the first time in 1894 in R. Dedekind's Supplement XI to the fourth edition of *Vorlesung über Zahlentheorie* by Lejeune Dirichlet and later in papers by F. S. Macauley, W. Krull, and E. Noether, depending on the fact that the ideals of a ring form a semiring. Semirings per se were at first considered explicitly in 1934 by H. S. Vandiver in connection with the axiomatization of natural and nonnegative rational numbers (cf. the excellent bibliography of about 600 references in the book by J. S. Golan). Later on semirings were investigated by numerous researchers in their own right in order to broaden techniques and to generalize results from ring theory or semigroup theory or in connection with some applications. However, semirings did not become as popular as one should expect with regard to their various applications, and the only attempts to present the algebraic theory of semirings as an integral part of modern algebra seems to be in the book *Algebra* by L. Rédei (1959/1967), in *Einführung in die Algebra, Arithmetik und Zahlen theorie*, by H. Lugowski and H. J. Weinert (material for corresponding study, published in Potsdam, 1961), and in Volume III of the *Cours d' Algèbre Générale* by Almeida Costa (1974). On the other hand, since semirings and also semimodules over them have become an important tool in applied mathematics and theoretical computer science, semirings appear—sometimes with other names such as, e.g., *dinoid*, *binoid*, or *valuation algebra*—in the literature of those fields with consistent and increasing frequency. Therefore, results on semirings and their applications are scattered through the literature and are not easily available to those who have to use them.

We repeat that, with a very few special exceptions, all semirings (or hemirings, etc.) $(S, +, \cdot)$ occurring in the literature satisfy at least the following axioms: $(S, +)$ and $(S, \cdot)$ are semigroups, and the multiplication distributes over addition from both sides. Very often $(S, +)$ is assumed to be commutative, which is justified by several reasons, not at least with respect to the use in ring theory (there are indeed also various "rings" with noncommutative addition). Other assumptions are that $(S, +)$ has a neutral element 0, or that $(S, \cdot)$ has a neutral element 1, or that the additive neutral 0 is multiplicatively absorbing, which means $0 \cdot a = a \cdot 0 = 0$ for all $a \in S$. In the book by J. S. Golan a hemiring $(S, +, \cdot)$ is defined by all these properties with the exception of the existence of a multiplicative neutral 1; and if such an element exists and differs from 0, $(S, +, \cdot)$ is called a semiring. This rather restrictive definition has the advantage that this kind of semiring is in fact the most important one with respect to numerous applications and that these assumptions prove useful for various statements on semirings which depend on some or all of them. On the other hand, a more general concept of a semiring must be used to include the various investigations on other semirings, where for instance those semirings occur which have no zero or for which such an element is not multiplicatively absorbing.

The book by J. S. Golan reviewed here is the first monograph on semirings to appear. It will be very helpful to everyone who works in this field or who needs statements on semirings for any application. In particular, the large number of examples of semirings and applications collected in this book with detailed references, many of them taken from recent works in all the different fields mentioned previously, is a most valuable contribution to make semirings more accessible. The first chapters, dealing with the structure of semirings as defined above, present among other topics the building of new semirings from old ones and the concepts of complemented elements, ideals, factor semirings, and morphisms of semirings and their kernels. In the following chapters Euclidean semirings and additively regular semirings are investigated. Chapters 13–17 are devoted to semimodules over semirings, including free, projective, and injective semimodules; the localization of semimodules; and linear algebra over semirings. Then partially ordered and lattice-ordered semirings are considered. The latter are defined as semirings $(S, +, \cdot)$ which are also lattices $(S, \vee, \wedge)$ such that $a + b = a \vee b$ and $ab \leq a \wedge b$ hold for all $a, b \in S$. Another important tool, in particular for applications, are infinite sums in semirings, defined in an abstract way and subjected to a set of axioms. If each family of elements of a semiring is summable, the semiring is called complete. Chapters 21 and 22 deal with complete lattice-ordered semirings and with fixed points of affine maps $\lambda: H \to M$, where M is a semimodule over a semiring.

In order to put so much material into about 280 pages of text, the presentation of Golan's monograph is concise and requires some effort by the reader. There are also a few minor errors in the mathematics. We mention that in autumn 1993 another book on semirings in German appeared, entitled *Halbringe—algebraische Theorie und Anwendungen in der Informatik* by U. Hebisch and H. J. Weinert. It contains much less material in a more detailed presentation.

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On topological and linear equivalence of certain function spaces, by J. A. Baars and J. A. M. de Groot. CWI Tract 86, Centrum Wisk. Inform., Amsterdam, 1992, 201 pp., DFL 60.00. ISBN 90 6196 411 3

The function spaces dealt with are spaces $\mathrm{C}(X)$ and $\mathrm{C}^*(X)$ of the respectively continuous and bounded continuous real-valued functions on a topological space X.

General topology says that, for every topological space X, there is a Hausdorff completely regular space (= Tychonov space) Y and a continuous surjection $\tau: X \to Y$ such that the mapping $T: \mathrm{C}(Y) \to \mathrm{C}(X)$, $f \mapsto f(\tau)$ is an isomorphism of $\mathrm{C}(Y)$ onto $\mathrm{C}(X)$, whose restriction to $\mathrm{C}^*(Y)$ is also an isomorphism of $\mathrm{C}^*(Y)$ onto $\mathrm{C}^*(X)$ (cf. [6, Theorem 3.9]). Therefore, one