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Fundamentals in the theory of operator algebras, III–IV. *An exercise approach*, by Richard V. Kadison and John R. Ringrose. Birkhäuser, Basel, III: 1991, xiv+273 pp., \$34.50. ISBN 0-8176-3497-5; IV: 1992, xiv+586 pp., \$49.50. ISBN 0-8176-3498-3

When Johann von Neumann in 1929 proved the double commutant theorem (the weak closure of a unital $*$-invariant algebra of operators on a Hilbert space is precisely the set of operators that commute with every operator commuting with the algebra), he set a train of thoughts in motion, whose destination even he could not have foreseen. The ensuing theory of operator algebras was originally motivated by the desire to create an axiomatic foundation for the emerging quantum theory, but during the long years of close collaboration between Johnny Neumann and Frank Murray things "got out of hand". The result was the monolith in twentieth century mathematics known as *Rings of operators*, I–IV. Respected by all, it was also daunting in its extreme technicality, and only a few dedicated mathematicians dared approach its summits.

Gelfand's work, which reached the West after World War II, resulted in the theory of abstract C^*-algebras and gave operator algebras a new direction, aimed at unitary representations of topological groups, mainly Lie groups. Although this multiplied the number of first-rank mathematicians working with operator algebras, the 1945–70 era may be labeled "the silent growth". Largely unnoticed by most of the mathematicial community and with only marginal interaction with other branches of mathematics, the theory gathered a number of powerful techniques and strong results.

The integration in mainstream mathematics and the recognition came in the seventies and culminated in the Fields medals awarded to Alain Connes (1982) and Vaughan Jones (1990). Now we see operator algebras used in geometry (foliations), PDE (index theorems), algebraic topology (K-theory and cyclic cohomology), and as the link between knot theory and models in statistical quantum mechanics. The theory has certainly come of age and has become fashionable. At the same time, the demand for concise textbooks that cover the subject has become noticeable, and this brings us to the books under review.

Fundamentals in the theory of operator algebras, Volumes I & II were published in 1982 and 1986. Since then they have quickly established themselves as The Textbooks in Operator Algebra Theory. To be sure, they do not match Dixmier's books in elegance, nor do they have the relentless drive in Sakai's slender volume; but, as the authors claim, they teach the subject. They do so in a sober, timeless manner, with respect for the student's intelligence and anticipation of his shortcomings.

One unavoidable consequence of the detailed, painstakingly accurate presentation in the Kadison-Ringrose volumes is the limited area they cover. The authors do not (by a long shot) tell all they know but give only the fundamentals. This is partially compensated by the generous number of exercises accompanying the text. The 715 exercises serve to illustrate and extend the results and examples in the text, but also help the reader to develop working techniques and facility with the subject matter. Some are routine, requiring nothing more than a clear understanding of a definition or a result for their solution. Other exercises (and groups of exercises) constitute small (guided) research projects. Anyone who claims to have solved them all is either boasting or is truly a master of our subject.

The present volumes, III and IV, contain the solutions of the exercises in the first two volumes, in what the authors feel to be optimal form (and few would quibble with that claim). This gives the student a model with which his own solutions can be compared and an indicator of the method and style for producing further solutions on an individual basis. For the more experienced and, maybe, impatient researcher it gives a speedy route through one or another of the many special topics that supplement those of the text proper of "Fundamentals". These topics include:

Algebras of affiliated operators
Approximate identities in C^*-algebras
β-compactification of $\mathbb{N}$
Canonical anticommutation relations
Characterizations of von Neumann algebras among C^*-algebras
Compact operators
Completely positive maps
Conditional expectations
The Connes invariant
Coupling constant and operator
Derivations and automorphisms
Diagonalization of operator-valued matrices
Dixmier approximation theorem
Examples of extreme points
Extremely disconnected spaces
Flip automorphisms
Friedrichs extension
The fundamental group of a II_1-factor
Generalized Schwarz inequality
Elements of harmonic analysis
Ideals in operator algebras
Isometries and Jordan homomorphisms
Modular theory
Nonnormal traces
Relative commutants
Representations
Stone-Weierstrass theorems for C^*-algebras
Strong continuity of operator functions
Tensor products of operator algebras
Unitary implementation of automorphisms
Unitary elements of C^*-algebras
Vectors and vector states

Presented in alphabetical order, as above, the list could give the reader a dizzying feeling of *embarras de richesse*; but, of course, each topic is presented in the book where it connects with the fundamental part of the theory. It is instructive, therefore, also to give the headings of the fourteen chapters that constitute volumes I and II:

Linear spaces
Basics of Hilbert space and linear operators
Banach algebras
Elementary C^*-algebra theory
Elementary von Neumann algebra theory
Comparison theory of projections
Normal states and unitary equivalence of von Neumann algebras
The trace
Algebra and commutant
Special representations of C^*-algebras
Tensor products
Approximation by matrix algebras
Crossed products
Direct integrals and decompositions

Many years ago Paul Halmos published a delightful book called *A Hilbert space problem book* that presented the elementary theory of operators in a series of problems (with hints and solutions). Appealing though this approach may be, it will probably not work in a highly technical field like operator algebras, where the teacher must step in from time to time to tell the student about heavy machinery that has to be developed before further progress can be made. Yet Halmos's dictum stands: *The only way to learn mathematics is to do mathematics.*

The completed four-volume treatise by Kadison and Ringrose seems to me to utilize the best of both methods: The fundamentals are explained as text to be read. The numerous exercises are inserted to challenge the curiosity, to develop "hands-on" skills, and to give a glimpse of wider spaces. Now the solutions, as in Halmos's book, appear at the end as the logical conclusion. The authors have erected a monument in mathematics in the tradition of Courant-Hilbert, Dunford-Schwartz, Hewitt-Ross, and Reed-Simon.

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An introduction to Γ-convergence, by Gianni Dal Maso, Birkhäuser, Boston, 1992, xiv+337 pp., $69.50. ISBN 0-8176-3679-X

Not long ago, a colleague at Courant Institute asked me: "Do you know a good reference on Γ-convergence and 'all that stuff'?" I realized that was not an easy question to answer. Although the topic has existed for over thirty years, I could not think of a single book or set of lecture notes that covered reasonably