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HENK BROER
UNIVERSITY OF GRONINGEN
E-mail address: h.w.broer@math.rug.nl

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Group theory and physics, by S. Sternberg. Cambridge University Press, Cambridge, 1994, xiii + 429 pp., $69.95. ISBN 0-521-24870-1

Textbooks for an elementary course in calculus or differential equations aimed at science and engineering students almost invariably stress the physical motivation underlying the subject matter. Examples and exercises naturally focus on the older and more commonly encountered parts of physics such as classical mechanics and electromagnetic theory. When the students are drawn from a wider range of disciplines such as economics and biology, the motivating examples have to be adjusted accordingly—and may become somewhat less persuasive due to artificial or oversimplified formulations.

The urge to justify mathematics to students by pointing to "real world" applications has led to more frequent inclusion of such topics as coding theory in recent textbooks on linear algebra and abstract algebra. But older books on algebra, or those written for graduate students, generally suppress any hint of applicability outside mathematics itself. In particular, most accounts of abstract group theory or group representations written by mathematicians ignore the rich applications of these subjects to twentieth century physics and chemistry. (One exception is the recent text [JL] on finite group representations by James and Liebeck, whose concluding chapter applies character theory to molecular vibration.)

Symmetry is an ever-present consideration in the study of natural phenomena. Group theory provides a precise language with which to describe the possible symmetries of a physical system. In the case of crystallographic groups, the successful classification of relevant symmetry groups in the nineteenth century made possible many predictions of what may actually be observed in nature. But modern physics sometimes requires more subtle data coming not just from the obvious geometric actions of groups but also from the possible representations of the groups by linear operators (typically acting on spaces of functions).

This is especially apparent in the study of elementary particles since the 1960s, leading to what is now known as the Standard Model. As this review is being written, news media are reporting confirmation by two research teams of the existence of the elusive (and massive) "top" quark. But neither the mass media nor the books by physicists intended for very general audiences give any clear impression of just what kind of mathematics is involved in the quark theory. Sternberg's book goes a long way in this direction, for a mathematically oriented reader.

This book is the outgrowth of years of seminar activity with both mathematicians and physicists and is directed toward nonspecialist readers in both communities. It is completely different from the careful didactic textbooks (such as [C] and [T]) used in graduate physics courses, which have the drawback of being unreadable by the average mathematician. Having taken only the obligatory undergraduate course in physics n years ago, I find that Sternberg opens up the subject to me in a helpful way—even though I still get lost in the more technical passages.

As he points out in his preface, the discovery of quantum mechanics led to a wider recognition of the importance of group representations (though resistance among physicists to the "Gruppenpest" lasted a long time). H. Weyl's 1928 book *Gruppentheorie und Quantenmechanik* treated group theory and physics in alternating chapters, making it too easy for physicists to skip the mathematics and for mathematicians to skip the physics. So the present book opts for a more integrated approach.

In five chapters (comprising 300 pages), Sternberg interweaves group theory and physics in engaging ways. Eight appendices occupy another 125 pages, ranging from a history of nineteenth century spectroscopy to accounts of Bravais lattices, formulas related to symmetric group representations, and Wigner's theorem on quantum mechanical symmetries. The appendices also contain more technical results on compact groups and characters of Lie groups.

The first chapter provides a gentle introduction to groups (both finite and compact) and group actions, with excursions into the role of groups in crystallography, the classification of finite subgroups of $O(3)$, and the interplay of the icosahedral group and Euler's formula with fullerenes ("buckyballs").

Next comes the classical representation theory of finite groups (characters and orthogonality relations), augmented by a treatment of the representations of symmetric groups in terms of Young diagrams. Physics plays a larger role in Chapter 3, where molecular vibrations occur as a motivating example for the use of group characters. Here the language of vector bundles and induced representations is brought into play in a nonthreatening way. Further technical tools are introduced (tensor products, semidirect products, Mackey theorems on induced representations), always in a suitable physical context. The Poincaré group and other Lie groups begin to play a major role, motivating the more systematic discussion of compact and noncompact Lie groups in Chapter 4. There the emphasis shifts to atomic physics: the hydrogen atom, the periodic table, relativistic wave equations.

Chapter 5 develops the Schur-Weyl theory of tensor representations of $GL(V)$, leading to the determination of irreducible finite-dimensional representations of special linear groups over $\mathbb{C}$. The payoff is an extensive discussion of the role played by some of this theory in elementary particle physics: strangeness, the eight-fold way (based on the 8-dimensional adjoint representation of $SU(3)$), quarks, color and beyond.

Sternberg's book is neither a technical treatise nor a textbook (there are no exercises). It is written in an informal conversational style, with only a few results stated formally. The index is somewhat unpredictable, including for example "buckyball spectra, 126" and "quark masses, 297", but omitting "buckyball, 45" and "fullerene, 43" and "quark, 288". There are quite a few misprints. More serious is the inadvertent omission of the bibliography (though there are helpful suggestions for further reading at the end of the book): mysterious citations in the text such as "see [WW80]" on page 301 lead nowhere. This is being repaired in the softcover reprinting now underway, according to the publisher.

Such faults aside, the book is the next best thing to having a long chat with the author about subjects which are obviously near to his heart. He has made a valuable contribution to the breaking down of artificial barriers between mathematics and physics.

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J. E. Humphreys
University of Massachusetts
E-mail address: jeh@math.umass.edu

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Green functions for second order parabolic integro-differential problems, by M. G. Garroni and J. L. Menaldi. Longman Scientific & Technical, Harlow, Essex, England, 1992, 417 pp., \$57.00. ISBN 3-582-02156-1

The topic of this book is the deep and highly involved treatment of parabolic equations of second order containing a nonlocal term of a special structure. This treatment is general in the sense that all types of boundary conditions are considered with equal weight, both in Hölder- and L_p-spaces.

The equation is $Lu - I(u) = f$, where

$$Lu = \frac{\partial u}{\partial t} - a_{ij}(x, t)\frac{\partial^2 u}{\partial x_i \partial x_j} - a_i(x, t)\frac{\partial u}{\partial x_i} - a_0(x, t)u$$

is a classical linear uniformly parabolic operator of second order with bounded $(\alpha, \frac{\alpha}{2})$-Hölder continuous coefficients. Chapter 1 collects all the standard material (Schauder estimates in $C_{\alpha,\alpha/2}$ and L_p, maximum principles, etc.) concerning L. The nonlocal operator $I(u)$, modelling jumps in the diffusion process, has (roughly) the structure

$$I(u) = \int_F [u(x+j(x, t, \xi), t) - u(x, t) - j(x, t, \xi)\cdot\nabla_x u(x, t)]m(x, t, \xi)\pi(d\xi)$$

with some σ-finite measure π on F, and $T_\theta(x) = x + \theta j(x, t, \xi)$ is a diffeomorphism for all $\theta \in [0, 1]$, t, ξ. The properties of I are discussed in Chapter 2; especially conditions are given such that $\|I(u)\|_\chi \le \varepsilon\|D^2 u\|_\chi + c_\varepsilon$ l.o.t. for $\chi = C_{\alpha,\alpha/2}$ or L_p. Due to the "minus" sign in front of $I(u)$ the maximum principle can also be saved; hence this would be enough for just solving the equation via fixpoint arguments.